AP Physics C: Electricity & Magnetism Review

AP-Physics C: Maxwell's Equations

1. **Gauss' Law** ($\oint E \bullet dA = \frac{q_{enc}}{\epsilon_o}$)
2. **Ampere's Law** ($\oint B \bullet ds = \mu_o I_{total}$)
3. **Gauss' Law for Magnetism** ($\oint B \bullet dA = 0$)
4. **Faraday's Law** ($\varepsilon = -\frac{d\Phi_B}{dt} = \oint E \bullet ds$)

Types of Error (in labs)

1. **System Error**

Materials, Air Resistance, Friction

2. **Mathematical Error**

Truncating, Calculation error

3. **Observational Error**

Parallax, Reaction time, measuring distance

UNIT 1: Electrostatics

Fundamentals of Unit 1 Physics

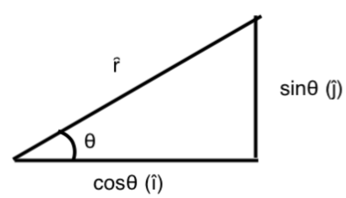
Atom : smallest particle of an element that maintains all the properties of that element
- Consists of:
    - Electron (negative charge)
    - Proton (positive charge)
    - Neutron (neutral charge)
- Neutral charge = equal number of electrons and protons
- Negative charge = more electrons than protons (electron excess)
- Positive charge = less electrons than protons (electron deficient)
- ONLY ADDING OR REMOVING ELECTRONS

Electron flow : flow of electrons between two objects until a charge equilibrium is reached, when charges on both objects are equal

Fundamental Charge Unit (FCU) : an amount of charge on one electron or one proton
- Macrocharge units are coulombs (C)
- 1 FCU = 1.6×10^{-19} C
- 1 FCU = 1 e
- e^- = electron, e^+ = proton

Atomic Mass Unit (AMU) : mass of a proton
- Macro mass units are kilograms (kg)
- 1 AMU = 1.67×10^{-27} kg

R hat (r̂) : unit vector in the radial direction. It always points away from the point charge regardless of the point charge's polarity.
- $+\hat{r}$ = radially outward
- $-\hat{r}$ = radially inward
- $\hat{r} = \cos\theta\ (\hat{i}) + \sin\theta\ (\hat{j})$
- $|\hat{r}| = \sqrt{\cos^2\theta\ (\hat{i}) + \sin^2\theta\ (\hat{j})} = 1$ (definition of a unit vector)



Opposite charges attract while like charges repel

Positive or negative charge doesn't indicate a positive or negative number, DO NOT use negative numbers in equations like Coulomb's Law. They only indicate the polarity of charge which is used to determine if there's repulsion or attraction.

Linear Charge Density (λ - Lambda) : constant LINEAR density for a uniformly charged distribution
- $\lambda = \frac{charge}{length} = \frac{Q}{L} = \frac{dq}{dx}$ [C/m]
- **<u>L</u>**ambda = **<u>L</u>**inear

Surface Charge Density (σ - sigma) : constant SURFACE density for a uniformly charged distribution

- $\sigma = \frac{charge}{area} = \frac{Q}{A} = \frac{dq}{dA}$ [C/m^2]
- **S**igma = **S**urface

Charging Objects

1. **Charging by friction (formication)**
    - Rubbing two materials together where the materials are on opposite sides of the triboelectric series (one material is positive while the other is negative)
        - Rubbing rabbit fur and PVC pipe

2. **Charging by conduction**

    Conductor Material Types:

    1. Conductors are usually metals (many free electrons)
    2. Insulators are usually nonmetals (no free electrons)
    3. Semiconductors (some free electrons, not important in Physics C: E&M)

    If a conducting object is charged by conduction, then the charges will be uniformly distributed on the surface due to free electrons allowing charges to move

    - Ex: Charging metal sphere with rod

    If an insulating object is charged by conduction, then the charges will be localized at the point of contact on the surface due to no free electrons for charges to move

    - Ex: Charging wooden sphere with rod

3. **Charging by induction**

    No physical contact is required between charged objects, mostly occurs on metals

    - Ex: Negatively charged rod near neutral ball connected to metal leaves will cause electrons in ball to flow towards leaves where they will repel each other

4. **Charging by polarization**

    Occurs mostly between nonmetals (insulators) and when atoms become polarized (one side has a partial positive charge while the other has a partial negative charge) due to electrons attracting to one side of an atom

    - Ex: Positively charged balloon near wall attracts electrons from the wall, causing surface atoms to be polarized which gives the atom a slight dipole. This allows the balloon to stick to the wall.

Law of Conservation of Charges (LCC)

Charges are never created, only separated

Coulomb's Law

The electrostatic force between 2 point charges is directly proportional to the product of the magnitude of each charge and inversely proportional to the square of the distance between them

- $F_e \propto q_1q_2$
- $F_e \propto 1 / r^2$
- $F_e \propto \frac{q_1q_2}{r^2}$
- $F_e = \frac{K_e q_1 q_2}{r^2}$
    - q_1 = point charge 1 [coulombs]
    - q_2 = point charge 2 [coulombs]
    - r = distance between point charges [meters]
    - K_e = electrostatic constant / coulomb's constant = 9 x 10^9 N m^2 / C^2
        - $K_e = \frac{1}{4\pi\varepsilon_o}$
        - ε_o [epsilon] = permittivity of free space or vacuum permittivity

Finding net force and equilibrium point of point charges

Net force for multiple point charges:

1. Understand which charges are attractive and which are repulsive
2. Create a free body diagram of the charge
3. Solve for net force, use components and unit vectors if necessary

Equilibrium point for a point charge between multiple point charges:

1. Understand where the point charge can physically be or not be
    a. Consider the directions of repulsive and attractive forces (have to balance out to 0)
    b. Doesn't always have to be between two (or more) point charges
2. Solve for distance by setting electrostatic forces equal to 0 in all directions
3. General Rule of Thumb: Equilibirum point will always be closer to smaller charge
    a. Small distance offsets the smaller charge to balance out greater charge

Applying FCU and AMU to prove an unintuitive idea

Background: Scientist claims that if two people standing at arms length (1m) had 1% greater charge, the electrostatic force between them is enough to lift the Earth

- Mass of Earth: 6 x 10^{24}
- FCU: 1.6 x 10^{-19} C
- AMU: 1.67 x 10^{-27} kg
- Each person weighs about 82 kg
- Distance between the persons: 1 m

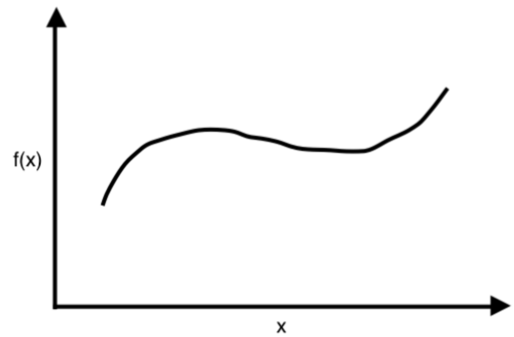
$$82\ kg \bullet \frac{1\ proton}{1.67\ x\ 10^{-27}\ kg} \bullet \frac{1.6\ x\ 10^{-19}\ C}{1\ proton} \bullet \frac{1\%}{100\%} = 3.92\ x\ 10^{7}\ C\ in\ each\ person$$

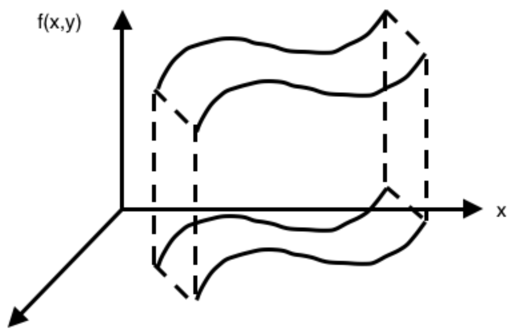
$$F_e = \frac{K_e q_1 q_2}{r^2} = \frac{(9\ x\ 10^9)(3.92\ x\ 10^7)(3.92\ x\ 10^7)}{1^2} = 1.389\ x\ 10^{25}\ N$$

F_e = ma → a = F_e / m

a = 1.389 x 10^{25} / 6 x 10^{24} = 2.315 m/s^2 (moving = lifted)

Electric Field or Electrostatic Field

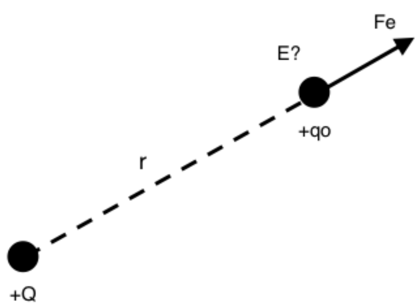




Field : JUST a function in higher dimensions

Electric Field / Electrostatic Field (E) [N / C] : a property of space around a particular charge (Q) in such a way that if a positive test charge (q_o) is placed near this original charge, the test charge will feel a force

- Test charge (q_o) is always positive
- Vector quantity denoted with E
- Measure of force per unit of test charge [N / C]

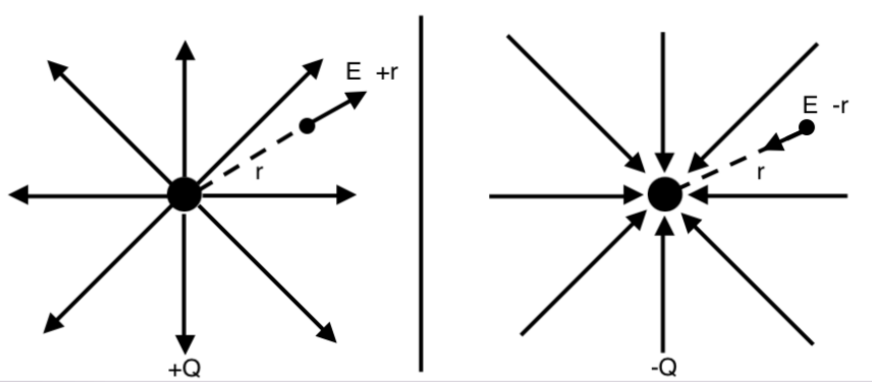


$F_e = \frac{K_e q_o Q}{r^2}$

$E = \frac{F_e}{q_o} = \frac{K_e Q}{r^2} (+\hat{r})$

- E = electric field due to a point charge Q at a distance r away from Q
- $\hat{r}$ = unit vector in direction of force
  - Would be $-\hat{r}$ if charge was -Q (attracted)

Drawing Electric Field Lines around a charge

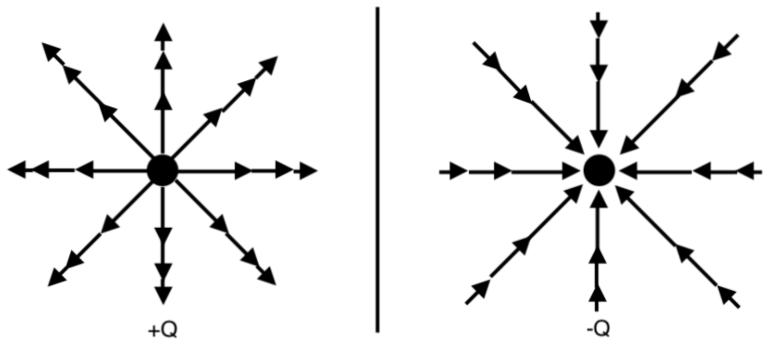


Electric field lines only show direction, not magnitude

Electric field lines points away from a positive charge

Electric field lines points inward toward a negative charge

Drawing Electric Field Vectors around a charge



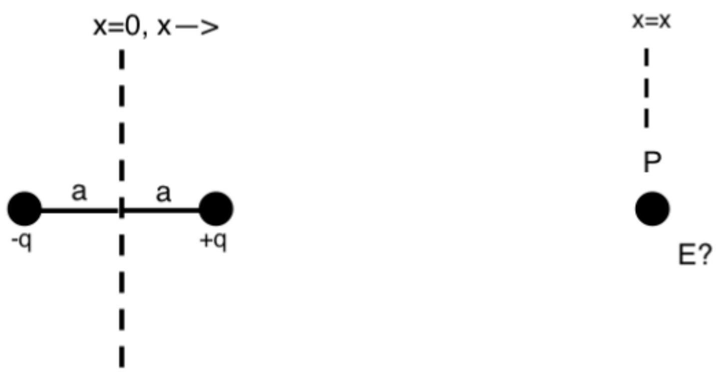
Electric field vectors show direction and magnitude

- Magnitude decreases as distance increases
- Magnitude varies at $1/r^2$

$+\hat{r}$ = radially outward $\quad$ $-\hat{r}$ = radially inward

Problem Solving: Electric Field due to electric dipole

1. **Solving for the electric field at a specific location (point P) when point P is located on the axis of the dipole**



E_+ = Electric field from +q $\quad$ E_- = Electric field from -q

$E_+ = \frac{K_e q}{(x-a)^2}(+\hat{i})$ $\quad$ $E_- = \frac{K_e q}{(x+a)^2}(-\hat{i})$ $\quad$ [establish E equations]

$E_p = E_+ + E_-$ [superimposing E]

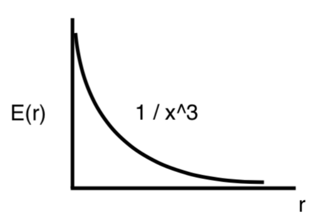
$E_p = \frac{K_e q}{(x-a)^2} - \frac{K_e q}{(x+a)^2}$ [substitute E_+ and E_-]

$E_p = \frac{4K_e qax}{(x^2-a^2)^2}(+\hat{i})$ [simplify with algebra]

($+\hat{i}$ makes sense because the repulsive electric field from +q is stronger as +q is closer to point P)

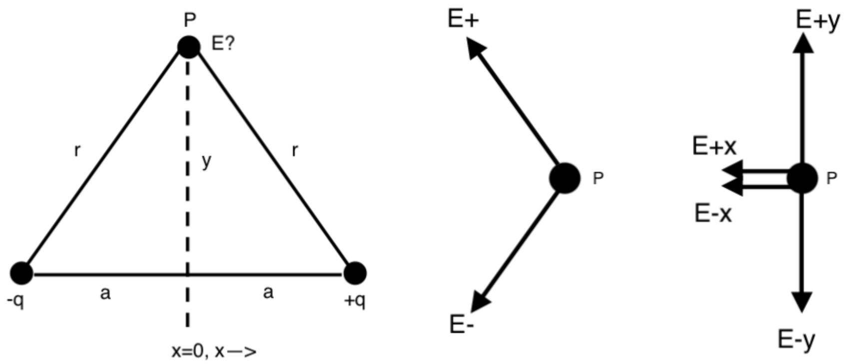
Extreme Case:
If x >> a, then $x^2 - a^2 = x^2$

$E_p = \frac{4K_e qax}{(x^2)^2} = \frac{4K_e qax}{x^4} = \frac{4K_e qa}{x^3}$



$E_p \propto \frac{1}{x^3}$ → Electric field varies by $\frac{1}{r^3}$ distance away from the electric dipole

2. **Solving for the electric field at a specific location (point P) when point P is located on the perpendicular bisector axis of the dipole**



y-components cancel
x-components superimposed (added) to find E_p

E_+ = Electric field from +q
$E_+ = \frac{K_e q}{r^2}$

E_- = Electric field from -q
$E_- = \frac{K_e q}{r^2}$

$r = \sqrt{a^2 + y^2}$
$\cos\theta = \frac{a}{r} = \frac{a}{\sqrt{a^2+y^2}}$

$E_p = E_+ + E_- = E_{+x} + E_{-x}$ [general equation]

$E_p = \frac{K_e q}{r^2}\cos\theta + \frac{K_e q}{r^2}\cos\theta\ (-\hat{i})$ [substitute x-component with cosine]

$E_p = \frac{K_e q}{r^2} * \frac{a}{\sqrt{a^2+y^2}} + \frac{K_e q}{r^2} * \frac{a}{\sqrt{a^2+y^2}}\ (-\hat{i})$ [substitute cosine with side ratios]

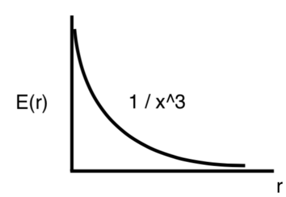
$E_p = \frac{2K_e qa}{(a^2+y^2)^{3/2}}\ (-\hat{i})$ [simplify with algebra]

(-î makes sense because there are only negative x-components from the electric field acting on point P)

Extreme Case:
If y >> a, then $a^2 + y^2 = y^2$

$E_p = \frac{2K_e qa}{(a^2+y^2)^{3/2}} = \frac{2K_e qa}{(y^2)^{3/2}} = \frac{2K_e qa}{y^3}$

$E_p \propto \frac{1}{y^3}$ → Electric field varies by $\frac{1}{r^3}$ distance away from the electric dipole



<u>Summary</u>
Steps:
1. Find all electric fields due to point charges acting on point P, use FBD if needed
2. Determine if any electric fields cancel out
3. Calculate the electric field from all point charges, may include trigonometry
4. Add all electric fields from all charges (superimposing)

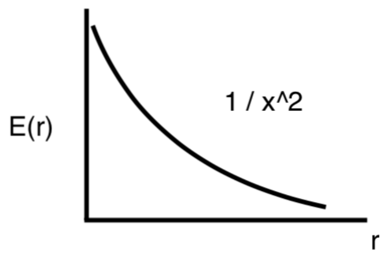
$E_p \propto \frac{1}{r^3}$ → Electric field varies by $\frac{1}{r^3}$ distance away from the electric dipole

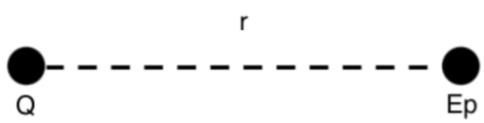
<table>
<tr><th>Point Charges</th><th>Electric Dipole
(2 point charges with opposite polarity)</th></tr>
<tr><td>

$|E(r)| = \frac{K_e q}{r^2}$

$E_p \propto \frac{1}{r^2}$

Electric field (E) varies at $\frac{1}{r^2}$





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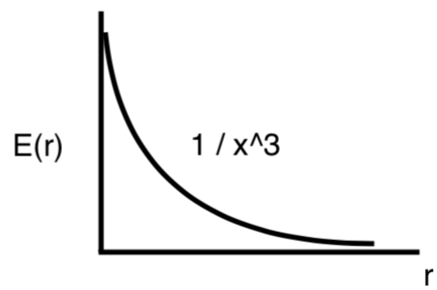
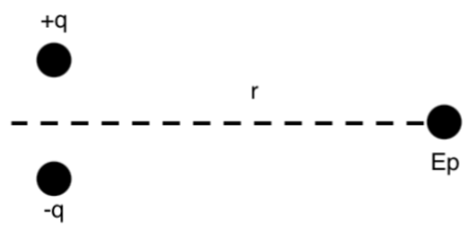
$|E(r)| = \frac{4K_e qa}{x^3}$ or $\frac{2K_e qa}{y^3}$ (look above)

$E_p \propto \frac{1}{r^3}$

Electric field (E) varies at $\frac{1}{r^3}$





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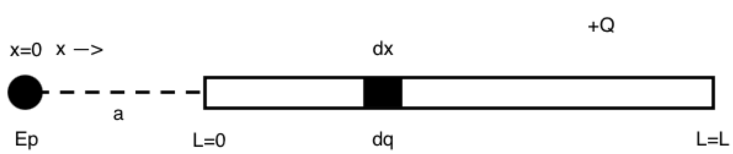
Problem Solving: Electric Field due to uniform charge distribution (rod)

1. **Solving for the electric field at a specific location (point P) when point P is located on the axis of the uniform charge distribution (rod)**



dx = small length of rod with dq charge
dq = point charge
Q = total charge of rod
E_p = electric field @ point p
dE_p = differential electric field @ point p
λ = linear charge density of rod [$\frac{charge}{length}$]
$\lambda = \frac{Q}{L} = \frac{dq}{dx} \rightarrow dq = \lambda dx$

$E_p = \frac{K_e q}{r^2}$ (-î) [general equation]

$dE_p = \frac{K_e dq}{x^2}$ (-î) [turn equation into differential]

$dE_p = \frac{K_e \lambda dx}{x^2}$ [substitute dq with lambda]

$\int_a^{a+L} dE_p = \int_a^{a+L} \frac{K_e \lambda}{x^2} dx$ [integrate dE_p with limits of rod]

$E_p = K_e\lambda\,(-x^{-1})\,|_a^{a+L}$ [evaluate with limits]

$E_p = K_e\lambda[-\frac{1}{a+L} + \frac{1}{a}]$ [substitute x with limits]

$E_p = \frac{K_e Q}{a(a+L)}\,(-\hat{\imath})$ [simplified with algebra]

Extreme Cases:

If a >> L, then a + L = a

$E_p = \frac{K_e Q}{a(a+L)} = \frac{K_e Q}{a(a)} = \frac{K_e Q}{a^2}$

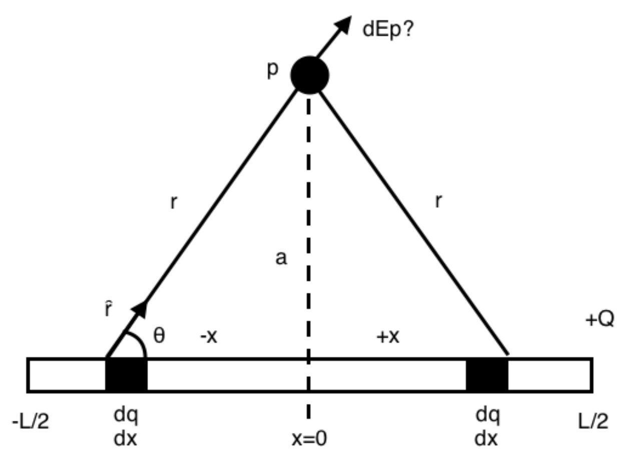
[proves Coulomb's Law, if distance is far from rod, rod appears as point charge]

If a << L, then a + L = L

$E_p = \frac{K_e Q}{a(a+L)} = \frac{K_e Q}{a(L)} = \frac{K_e Q}{aL}$

[doesn't prove Coulomb's Law but maintains meters^2 in denominator, units stay consistent]

2. **Solving for the electric field at a specific location (point P) when point P is located on the perpendicular bisector axis of the uniform charge distribution (rod)**



dx = small length of rod with dq charge

dq = point charge

Q = total charge of rod

dE_p = differential electric field @ point p

λ = linear charge density of rod $[\frac{charge}{length}]$

$\lambda = \frac{Q}{L} = \frac{dq}{dx} \rightarrow dq = \lambda dx$

$\hat{r}$ = direction of dE_p

$\hat{r} = \cos\theta\,(\hat{\imath}) + \sin\theta\,(\hat{\jmath})$

$|\hat{r}| = \sqrt{\cos^2\theta\,(\hat{\imath}) + \sin^2\theta\,(\hat{\jmath})} = 1$

$E_p = \frac{K_e q}{r^2}\,(\hat{r})$ [general equation]

$dE_p = \frac{K_e dq}{r^2}\,(\hat{r})$ [turn E_p into differential]

$dE_p = \frac{K_e dq}{r^2}\cos\theta\,(\hat{\imath}) + \frac{K_e dq}{r^2}\sin\theta\,(\hat{\jmath})$ [convert $\hat{r}$ into $\hat{\imath}$ and $\hat{\jmath}$ or x and y]

$dE_p = \frac{K_e \lambda dx}{x^2+a^2}\cos\theta\,(\hat{\imath}) + \frac{K_e \lambda dx}{x^2+a^2}\sin\theta\,(\hat{\jmath})$ [substitute dq and r^2]

$dE_p = \frac{K_e \lambda dx}{x^2+a^2} * \frac{-x}{\sqrt{x^2+a^2}}\,(\hat{\imath}) + \frac{K_e \lambda dx}{x^2+a^2} * \frac{a}{\sqrt{x^2+a^2}}\,(\hat{\jmath})$ [convert $\cos\theta$ and $\sin\theta$ into ratios]

$dE_p = -\frac{K_e\lambda x dx}{(x^2+a^2)^{3/2}}$ (î) + $\frac{K_e\lambda a dx}{(x^2+a^2)^{3/2}}$ (ĵ) (differential for E_p at location p for a charged rod)

$\int_{-L/2}^{L/2} dE_p = -\int_{-L/2}^{L/2} \frac{K_e\lambda x dx}{(x^2+a^2)^{3/2}}$ (î) + $\int_{-L/2}^{L/2} \frac{K_e\lambda a dx}{(x^2+a^2)^{3/2}}$ (ĵ) [integrate and applying limits]

x-component	y-component
$E_{px} = -\int_{-L/2}^{L/2} \frac{K_e\lambda x dx}{(x^2+a^2)^{3/2}}$ (î) [general eq.]	$E_{py} = \int_{-L/2}^{L/2} \frac{K_e\lambda a dx}{(x^2+a^2)^{3/2}}$ (ĵ) [general eq.]
$E_{px} = -K_e\lambda \int_{-L/2}^{L/2} \frac{x}{(x^2+a^2)^{3/2}} dx$ (î) [move constants]	$E_{py} = K_e\lambda a \int_{-L/2}^{L/2} \frac{dx}{(x^2+a^2)^{3/2}}$ (ĵ) [move constants]
$E_{px} = -\frac{K_e\lambda}{2}\int \frac{1}{u^{3/2}} du$ [u-sub, $u = x^2 + a^2$]	$E_{py} = K_e\lambda a[\frac{x}{a^2(x^2+a^2)^{1/2}}]_{-L/2}^{L/2}$ [integrate]
$E_{px} = -\frac{K_e\lambda}{2} * \frac{-2}{u^{1/2}}$ [integrate and simplify]	$E_{py} = \frac{K_e\lambda L}{a(\frac{L^2}{4}+a^2)^{1/2}}$ [apply limits and simplify]
$E_{px} = \frac{K_e\lambda}{\sqrt{x^2+a^2}}\Big\|_{-L/2}^{L/2}$ [sub for u and reapply limits]	$E_{py} = \frac{K_e Q}{a(\frac{L^2}{4}+a^2)^{1/2}}$ (ĵ) [sub lambda for Q/L]
$E_{px} = 0$ [simplify with algebra and solve]	

$E_p = E_{py} = E_{py} = \frac{K_e Q}{a(\frac{L^2}{4}+a^2)^{1/2}}$ (ĵ)

Extreme Cases:

If a >> L, then $\frac{L^2}{4} + a^2 = a^2$

$E_p = \frac{K_e Q}{a(\frac{L^2}{4}+a^2)^{1/2}} = \frac{K_e Q}{a(a^2)^{1/2}} = \frac{K_e Q}{a^2}$

[proves Coulomb's Law]

If a << L, then $\frac{L^2}{4} + a = \frac{L^2}{4}$

$E_p = \frac{K_e Q}{a(\frac{L^2}{4})^{1/2}} = \frac{K_e Q}{a(L/2)} = \frac{2K_e Q}{aL} = \frac{2K_e\lambda}{a}$ (all constants)

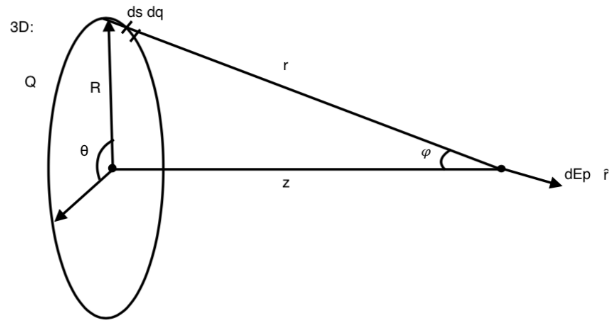
[doesn't prove Coulomb's Law, if close to rod then E_p stays constant]

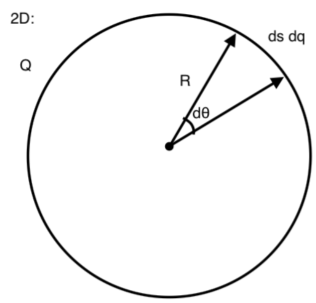
Summary

Steps:

1. Convert $E_p = \frac{K_e q}{r^2}$ into a differential and substitute with linear charge density (λ)
2. If dE_p is in r̂ direction, convert to x (î) and y (ĵ) using sinθ and cosθ
    a. If you have side lengths, change sinθ and cosθ into respective ratios
3. Integrate dE_p and apply limits to get E_p
    a. May require U-Substitution, magic table, and combining fractions

Problem Solving: Electric Field due to uniform charge distribution (ring)





$\lambda = \frac{Q}{2\pi R} = \frac{dq}{ds}$

$dq = \lambda ds$

$ds = R d\theta$

$dq = \lambda R d\theta$

$r = \sqrt{z^2 + R^2}$

Realize:

- X and Y components will cancel while only z-components will survive
- E_p will be 0 at the center of the ring, 0 distance away from the center along the ring's axis (z=0)

$E_p = \frac{K_e q}{r^2} (\hat{r})$ [general equation]

$dE_p = \frac{K_e dq}{r^2} (\hat{r})$ [turn E_p into differential]

$dE_p = \frac{K_e dq}{r^2} \cos\varphi\ (\check{k})$ [convert $\hat{r}$ into $\check{k}$ or z]

$dE_p = \frac{K_e \lambda R d\theta}{z^2+R^2} \cos\varphi\ (\check{k})$ [convert dq and r^2]

$dE_p = \frac{K_e \lambda R d\theta}{z^2+R^2} * \frac{z}{\sqrt{z^2+R^2}} (\check{k})$ [convert $\cos\varphi$ into ratio]

(differential eq. for E_p at location p away from ring's center along its axis for a charged ring)

$dE_p = \frac{K_e \lambda R z d\theta}{(z^2+R^2)^{3/2}} (\check{k})$

$E_p = \int_0^{2\pi} dE_p = K_e \lambda R z \int_0^{2\pi} \frac{d\theta}{(z^2+R^2)^{3/2}} (\check{k})$ [integrate, apply limits around ring, and move constants]

$E_p = \frac{2\pi K_e \lambda R z}{(z^2+R^2)^{3/2}} (\check{k})$ [evaluate]

$E_p = \frac{K_e Q z}{(z^2+R^2)^{3/2}} (\check{k})$ [simplify and replace lambda]

Extreme Cases:

If z == 0, then location at center of ring

$E_p = \frac{K_e Q z}{(z^2+R^2)^{3/2}} = 0$ N/C

[proves $E_p = 0$ at center of ring]

If z >> R, then $z^2 + R^2 = z^2$, ring is essentially a point charge

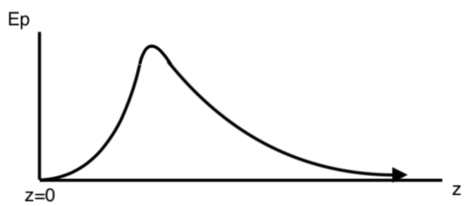
$E_p = \frac{K_e Q z}{(z^2+R^2)^{3/2}} = \frac{K_e Q z}{(z^2)^{3/2}} = \frac{K_e Q z}{z^3} = \frac{K_e Q}{z^2}$

[proves Coulomb's Law as ring is essentially a point charge]

Summary

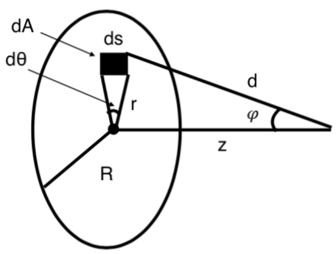
Steps:

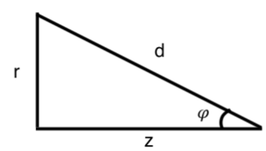
1. Convert $E_p = \frac{K_e q}{r^2}$ into a differential and substitute with linear charge density (λ)
2. If dE_p is in $\hat{r}$ direction, convert to z ($\check{k}$) using $\cos\varphi$ (or surviving components only)
3. Integrate dE_p and apply limits of the entire ring (0 to 2π) to get E_p
    b. May require U-Substitution, magic table, and combining fractions

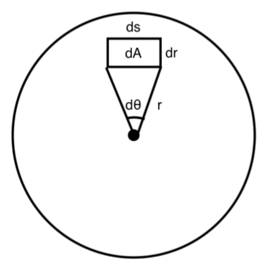


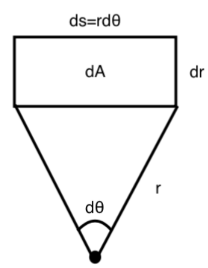
- E_p doesn't always decrease as location P gets farther from the center of the ring/disk
- Graph represents how E_p varies as a function of z (distance from the center of ring/disk)

Problem Solving: Electric Field due to uniform charge distribution (disk)









(converting $\hat{r}$)

$d = \sqrt{z^2 + r^2}$

$\cos\varphi = \frac{z}{d} = \frac{z}{\sqrt{z^2 + r^2}}$

(density relationship)

$\sigma = \frac{Q}{\pi R^2} = \frac{dq}{dA} = \frac{dq}{r d\theta dr}$

$dq = \sigma r d\theta dr$

(area relationship)

A = b * h

$dA = r d\theta dr$

Realize:

- X and Y components will cancel while only z-components will survive
- E_p will be 0 at the center of the disk, 0 distance away from the center along the disk's axis (z=0)

$E_p = \frac{K_e q}{r^2}(\hat{r})$ [general equation]

$dE_p = \frac{K_e dq}{r^2}(\hat{r})$ [turn E_p into differential]

$dE_p = \frac{K_e dq}{r^2}\cos\varphi\,(\check{k})$ [convert $\hat{r}$ into $\check{k}$ or z]

$dE_p = \frac{K_e \sigma r d\theta dr}{z^2 + r^2}\cos\varphi\,(\check{k})$ [convert dq and r^2]

$dE_p = \frac{K_e \sigma r d\theta dr}{z^2 + r^2} * \frac{z}{\sqrt{z^2 + r^2}}(\check{k})$ [convert $\cos\varphi$ into ratio]

(differential eq. for E_p at location p away from ring's center along its axis for a charged disk)

$dE_p = \frac{K_e \sigma z d\theta dr}{(z^2+r^2)^{3/2}}$ $(\check{k})$

$E_p = \int_0^R \int_0^{2\pi} dE_p = K_e \sigma z \int_0^R \int_0^{2\pi} \frac{r d\theta dr}{(z^2+r^2)^{3/2}}$ $(\check{k})$ [apply limits around disk and move constants]

$E_p = K_e \sigma z \int_0^R \frac{rdr}{(z^2+r^2)^{3/2}} \int_0^{2\pi} d\theta$ $(\check{k})$ [split integrals to handle respective domains]

$E_p = K_e \sigma z \int_0^R \frac{rdr}{(z^2+r^2)^{3/2}} (2\pi) = 2\pi K_e \sigma z \int_0^R \frac{rdr}{(z^2+r^2)^{3/2}}$ $(\check{k})$ [evaluate θ integral and move constant (2π)]

$E_p = 2\pi K_e \sigma z [\frac{-1}{(z^2+r^2)^{1/2}}]_0^R$ $(\check{k})$ [integrate r integral and apply limits]

$E_p = 2\pi K_e \sigma z [\frac{-1}{(z^2+R^2)^{1/2}} + \frac{1}{z}]$ $(\check{k})$ [evaluate r integral]

$E_p = 2\pi K_e \sigma z (\frac{\sqrt{z^2+R^2} - z}{z\sqrt{z^2+R^2}})$ $(\check{k})$ [simplify with algebra]

Extreme Cases:

If z == 0, then at location P at center of disk

$E_p = 2\pi K_e \sigma z (\frac{\sqrt{z^2+R^2} - z}{z\sqrt{z^2+R^2}}) = 0$ N/C $(\check{k})$

[proves $E_p = 0$ at center of disk]

If z >> R, then $z^2 + R^2 = z^2$, disk is essentially a point charge

$E_p = 2\pi K_e \sigma z (\frac{\sqrt{z^2+R^2} - z}{z\sqrt{z^2+R^2}}) = 2\pi K_e \sigma z (\frac{\sqrt{z^2} - z}{z\sqrt{z^2}}) = 2\pi K_e \sigma z (\frac{z-z}{z^2})$ $(\check{k})$

$E_p = \frac{K_e Q}{z^2}$ $(\check{k})$

[proves Coulomb Law]

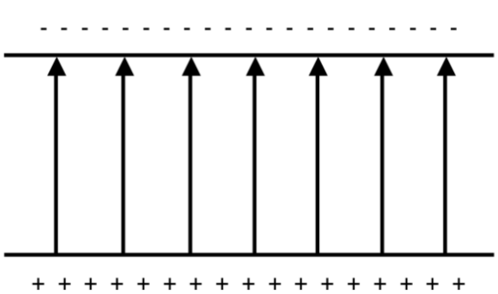
<u>Summary</u>

Steps:

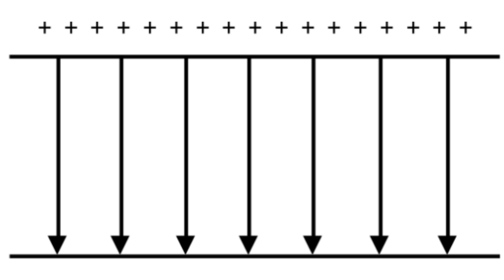
1. Convert $E_p = \frac{K_e q}{r^2}$ into a differential and substitute with linear charge density (λ)
2. If dE_p is in $\hat{r}$ direction, convert to z $(\check{k})$ using cosφ (or surviving components only)
3. Integrate dE_p twice for dθ and dr and apply limits of the entire disk (0 to 2π and 0 to R) to get E_p
    c. May require U-Substitution, magic table, and combining fractions

<u>Applications of Electric Field</u>

Motion of charged particles in a uniform electric field which are created using infinite sheets of charges



Uniform E-Field
[Capacitor]



Use kinematics to find V_o, V_f, a, t, and Δx of a particle beamed into a uniform electric field.

- Remember:
  - $E = F_e/q$
  - $F_e = Eq = ma$

TIPS ON HOW TO SOLVE PROBLEMS:

1. IMPORTANT EQUATIONS/CONSTANTS
   a. $F_e = \frac{K_e q_1 q_2}{r^2}$
   b. $E_p = \frac{K_e q}{r^2}$
   c. $\hat{r} = \cos\theta\,(\hat{\imath}) + \sin\theta\,(\hat{\jmath})$
   d. $\lambda = \frac{charge}{length} = \frac{Q}{L} = \frac{dq}{dx}$
   e. $dE_p = -\frac{K_e \lambda x dx}{(x^2+a^2)^{3/2}}(\hat{\imath}) + \frac{K_e \lambda a dx}{(x^2+a^2)^{3/2}}(\hat{\jmath})$ [differential eq. ONLY for uniformly charged rod]
   f. $K_e = 9 \times 10^9$ N m^2 / C^2
   g. FCU: 1.6×10^{-19} C
   h. AMU: 1.67×10^{-27} kg
2. List of useful integrals: (a = constant)
   a. $\int \frac{x}{(x^2+a^2)^{3/2}}dx = \frac{1}{(x^2+a^2)^{1/2}}$ (used in bisector axis of charged rod in x-direction)
   b. $\int \frac{dx}{(x^2+a^2)^{3/2}} = \frac{x}{a^2(x^2+a^2)^{1/2}}$ (used in bisector axis of charged rod in y-direction)
   c. $\int \frac{rdr}{(a^2+r^2)^{3/2}} = \frac{-1}{(a^2+r^2)^{1/2}}$ (used in charged disk)
3. General Process for problem-solving electric field questions at a location (P)
   a. Determine the direction of E_p conceptually (this can rule out directions that cancel)
   b. Determine density relationships and any trigonometry required like sin/cos
   c. Differentiate E_p to replace dq and r^2 (sometimes you might not need to)
   d. Change $\hat{r}$ into directions where E_p exists using sin/cos and trigonometric relationships from step b
   e. Combine, multiply, and simplify to obtain differential equation
   f. Integrate within respective domains, may have 2+ integrals for each domain (see disk example)
   g. Simplify with algebra
   h. If prompted, substitute the variable with the value given
4. If the sides of a triangle are known, don't solve for θ and use sine and cosine
   a. Remember sinθ = opposite / hypotenuse
   b. Remember cosθ = adjacent / hypotenuse
5. Positive or negative charge doesn't indicate a positive or negative number, DO NOT use negative numbers in equations like Coulomb's Law. They only indicate the polarity of charge which is used to determine if there's a repulsion or attraction.
6. Equilibrium point will always be closer to smaller charge

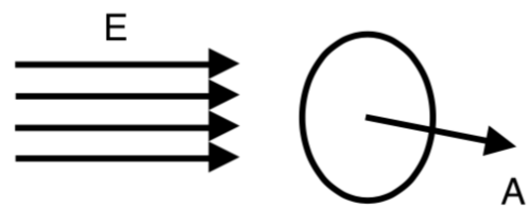
    a. Small distance offsets the smaller charge to balance out the greater charge
7. Extreme case is only when some variable is much larger than another variable which makes addition/subtraction obsolete, it is NOT the same as when the variable equals a set value
8. Solving particles in uniform electric fields is practically solving projectile motion problems with kinematics

UNIT 2: Electrostatics (Gauss' Law)

Fundamentals of Unit 2 Physics

Vector Flux (Φ - Phi) : a scalar quantity of a vector field that is passing through a surface

Electric Field Flux(Φ - Phi) [N m^2 / C] : a scalar quantity of an electric field that is passing through a surface



- Dot product between Electric Field and Area
    - **Φ** = E * A = |E||A|cosθ
        - θ = angle between E and A
- Flux depends on:
    - Magnitude of E
        - Bigger field = larger flux
    - Area of the surface
        - Larger area = larger flux
    - Angle between area vector and electric field vector
        - More perpendicular = larger flux
- Flux due to a constant electric field through a closed surface (sphere, cube, cylinder) is ALWAYS 0

Area vector : perpendicular vector to a surface's plane, similar concept to area under the curve (integral)
- Always points outward on a closed surface

Electrostatic Constant / Coulomb's Constant (K_e) : universal constant
- $K_e = \frac{1}{4\pi\varepsilon_o} = 9 \times 10^9$ N m^2 / C^2

Permittivity of Free Space / Vacuum Permittivity (ε_o - Epsilon) : numerical difficulty of generating, maintaining, propagating an electric field in free space OR how much charge is permitted on the surface of a conductor if that conductor is placed in space
- $\varepsilon_o = 8.85 \times 10^{-12}$ C^2 / N m^2

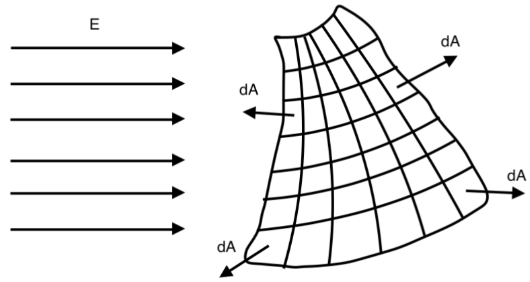
Definite Surface Integral (∮) : integral across an entire surface
- Part of Calculus 3 so won't actually do the integral
- Used for Gauss' Law and Gaussian surfaces

Volumetric Charge Density (ρ - Rho) : charge per unit of volume [C/m^3]
- Occurs within an insulating object as charge is distributed throughout volume, not just surface
- A conducting object (metal) will have charge distributed throughout surface only

Conducting object vs insulating object :
- Conducting means the charge is distributed throughout the surface of the object (metal)
- Insulating means the charge is distributed throughout the volume of the object (wood)

- Deals with ϱ and $\varrho(r)$ if charge is not uniformly distributed

Electric Flux due to a non-uniform surface



dA = differential area element
E = electric field
∯ = definite surface integral

$\Phi_E = \oint E \bullet dA = \frac{q_{enc}}{\varepsilon_o}$

Gauss' Law

Electric field flux through a closed surface equals to the magnitude of enclosed charge divided by the permittivity of free space (vacuum)

- Purpose of Gauss' Law = easier to find electric fields at certain locations compared to Coulomb's Law
- $\Phi_E = \oint E \bullet dA = \frac{q_{enclosed}}{\varepsilon_o}$

When to apply Gauss' Law:

- When things are symmetric
- When things are infinite

To apply Gauss' Law:

1. Put a gaussian surface around the charge
   a. Sphere, cylinder, rectangular prism (Pill Box)
2. Place the charge or charge distribution in a systematic manner relative to the gaussian surface
   a. Pretty much: Place at symmetrical center so E is equal at all points of the surface
3. Apply Gauss' Law over your chosen gaussian surface

Problem-Solving Steps:

- Right side:
  1. Figure out relationship between density ($\lambda/\sigma/\varrho$) and $q_{enclosed}$
     a. Ensure the right variables are substituted to match the space of the enclosed charge (similar to Left Side: Step 6 and examples in case study 6 and 7)
     b. When solving for q_{enc}, make sure you substitute A (surface area) or V (volume) with the right variable (r or R) to match the actual enclosed charge
        i. Use r when inside object and R when outside object
     c. $q_{enc} = \sigma A$ (conducting object)
     d. $q_{enc} = \varrho V$ (insulating object)
     e. $q_{enc} = \int_0^r \varrho(r)\, SA\, dr$ (insulating object but not uniformly distributed charge)
  2. Substitute for $q_{enclosed}$ to solve right side of Gauss' Law
- Left side:

1. Start with Gauss' Law general equation
2. Split integrals to all sides of the gaussian surface (top/bottom/sides)
3. Cancel out surfaces with no electric flux and explain why (because $E \perp A$)
4. Remove dot product operator and explain why (because $E \parallel A$)
5. Move E out of integral and explain why (E is constant because of centered charge or infinite distribution)
6. Substitute for $\oint dA$ (surface area for GAUSSIAN SURFACE)
7. Solve for E with algebra

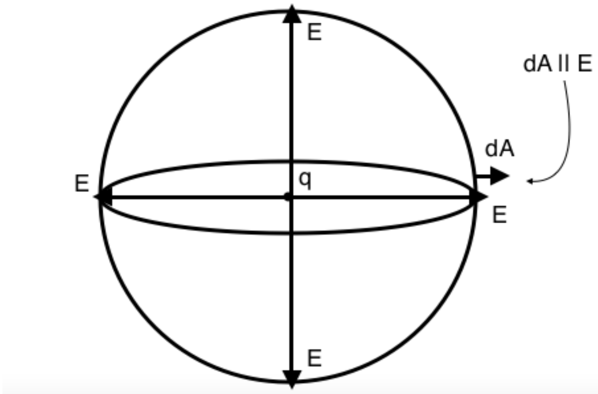
<u>Gaussian Surfaces</u>

- Sphere = $4\pi r^2$
- Cylinder body = $2\pi rL$
- Cylinder cap = πr^2
- Rectangular Prism (Pill Box) = $6L^2$

<u>Problem-Solving: List of Gauss' Law Problems</u>

1. Point charge
2. An infinite line of charge
3. Uniformly distributed non-conducting infinite sheet of charge
4. Infinite sheet of charge conducting
5. Conducting solid sphere (inside and outside)
6. Conducting infinite rod (inside and outside or a hollow rod)
7. Uniformly distributed non-conducting solid sphere (inside and outside)
    a. What if it was hollow inside?
8. Uniformly distributed non-conducting infinite solid rod (inside and outside)
    a. What if it was hollow inside?
9. Non-conducting, non-uniformly distributed charge on an infinite rod (inside and outside)
10. Non-conducting, non-uniformly distributed charge on a sphere (inside and outside)

1. Applying Gauss' Law for a point charge



$\oint E \bullet dA = \frac{q_{enc}}{\varepsilon_o}$

- Since E always parallel to dA at surface of sphere because charge is centered, we can ignore dot product ($\cos 0^\circ = 1$)

$\oint EdA = \frac{q_{enc}}{\varepsilon_o}$

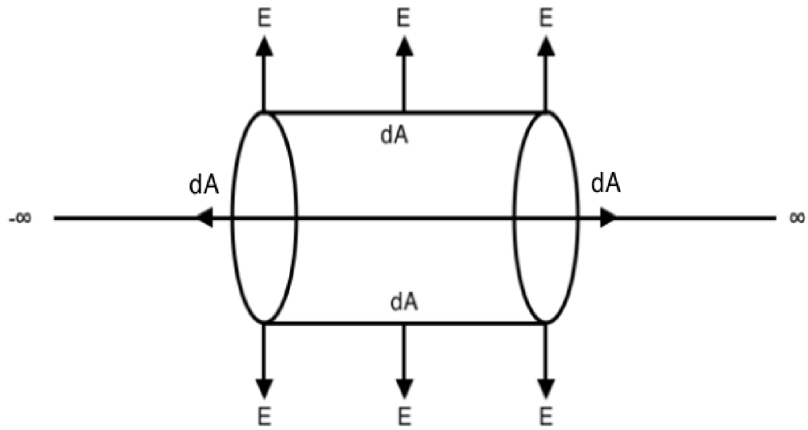
- E is constant because charge is centered within the sphere

$E \oint dA = \frac{q_{enc}}{\varepsilon_o}$

$E * 4\pi r^2 = \frac{q_{enc}}{\varepsilon_o}$

$E = \frac{K_e q_{enc}}{r^2} \hat{r}$ [proves coulomb's law]

2. Applying Gauss' Law for an infinite line of charge (thin charged rod)



$\lambda = \frac{q_{enc}}{L}$ $q_{enc} = \lambda L$

L = length of cylinder

Remember: $K_e = \frac{1}{4\pi\varepsilon_o}$

$\oint E \bullet dA = \frac{q_{enc}}{\varepsilon_o}$

- 2 integrals to account for 2 cylinder caps and cylinder body

$2\oint E \bullet dA$ (cylinder caps) + $\oint E \bullet dA$ (cylinder body) = $\frac{\lambda L}{\varepsilon_o}$

- E is perpendicular to dA of cylinder caps so it cancels (cos90˚ = 0)

$\oint E \bullet dA$ (cylinder body) = $\frac{\lambda L}{\varepsilon_o}$

- Since E always parallel to dA at surface of cylinder body because charge is infinitely distributed, we can ignore dot product (cos0˚ = 1)

$\oint EdA = \frac{\lambda L}{\varepsilon_o}$

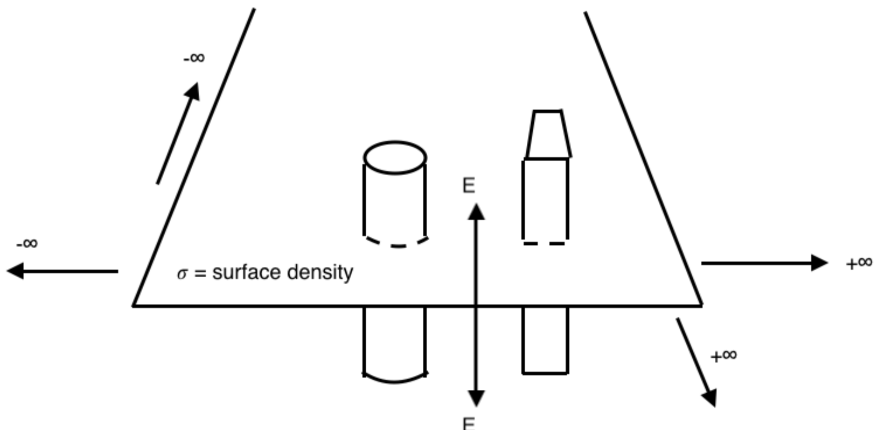
- E is constant because charge is centered within the cylinder

$E\oint dA = \frac{\lambda L}{\varepsilon_o}$

$E * 2\pi rL = \frac{\lambda L}{\varepsilon_o}$

$E = \frac{2K_e\lambda}{r}\hat{r}$

3. Applying Gauss' Law for a uniformly distributed infinite charged plane (non-conducting sheet of charge)



$\sigma = \frac{Q}{A} = \frac{dq}{dA}$

$q_{enc} = \sigma dA = \sigma A$

dA = cross-section of gaussian surface

$\oint E \bullet dA = \frac{q_{enc}}{\varepsilon_o}$

- 3 integrals to account for top/bottom/sides of pillbox or cylinder

$\oint E \bullet dA \text{ (top)} + \oint E \bullet dA \text{ (bottom)} + \oint E \bullet dA \text{ (side)} = \frac{\sigma A}{\varepsilon_o}$

- Side of pillbox/cylinder generate no flux because E is perpendicular to dA

$\oint E \bullet dA \text{ (top)} + \oint E \bullet dA \text{ (bottom)} = \frac{\sigma A}{\varepsilon_o}$

- Top and bottom of either gaussian shape will be the same

$2\oint E \bullet dA = \frac{\sigma A}{\varepsilon_o}$

- Since E always parallel to dA at top/bottom surface of pillbox/cylinder because charge is infinitely distributed, we can ignore dot product ($\cos 0° = 1$)

$2\oint E dA = \frac{\sigma A}{\varepsilon_o}$

- E is constant because charge is infinitely distributed

$2E\oint dA = \frac{\sigma A}{\varepsilon_o}$

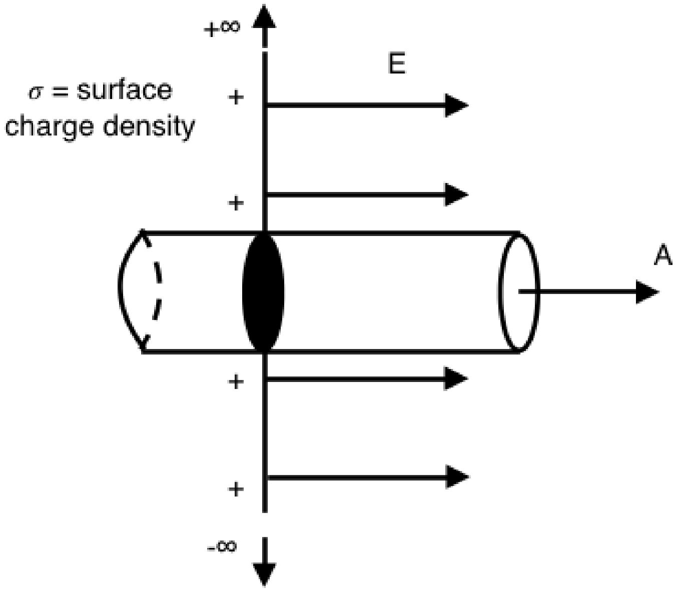
$2EA = \frac{\sigma A}{\varepsilon_o}$

$E = \frac{\sigma}{2\varepsilon_o}\hat{r}$

4. Applying Gauss' Law for a uniformly distributed infinite charged metal plate

$\sigma = \frac{Q}{A} = \frac{dq}{dA}$ $q_{enc} = \sigma dA = \sigma A$

dA = cross-section of gaussian surface



$\oint E \bullet dA = \frac{q_{enc}}{\varepsilon_o}$

- 2 integrals to account for cap/sides of cylinder, left cap ignored because no charge inside metal

$\oint E \bullet dA$ (side) + $\oint E \bullet dA$ (cap) = $\frac{\sigma A}{\varepsilon_o}$

- Side of pillbox/cylinder generate no flux because E is perpendicular to dA

$\oint E \bullet dA$ (cap) = $\frac{\sigma A}{\varepsilon_o}$

- Since E always parallel to dA at cap surface of cylinder because charge is infinitely distributed, we can ignore dot product ($\cos 0^\circ = 1$)

$\oint EdA = \frac{\sigma A}{\varepsilon_o}$

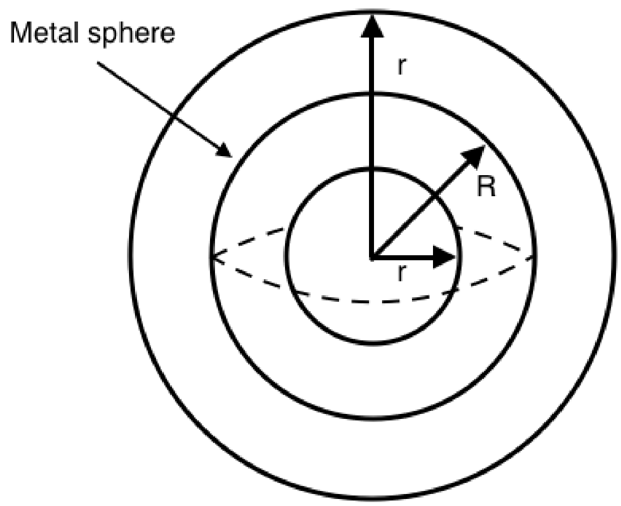
- E is constant because charge is infinitely distributed

$E \oint dA = \frac{\sigma A}{\varepsilon_o}$

$EA = \frac{\sigma A}{\varepsilon_o}$

$E = \frac{\sigma}{\varepsilon_o} \hat{r}$

5. Applying Gauss' Law for a conducting metal sphere of charge Q (inside and outside)



q_{enc} = 0 when r < R

q_{enc} = Q when r > R

dA = cross-section of gaussian surface

Note: Inside of a metal, the static electric field is 0, this isn't true if there's a current flowing through the metal as there will be a flowing electric field, AKA Faraday's Cage Phenomenon

r < R (inside metal sphere)

$\oint E \bullet dA = \frac{q_{enc}}{\varepsilon_o}$

- Since we are inside of the metal sphere and all the charge is located on the surface, there is no q_{enc} or $q_{enc} = 0$

E(r) = 0 when r < R

r > R (outside metal sphere)

$\oint E \bullet dA = \frac{q_{enc}}{\varepsilon_o}$

- Since E always parallel to dA at surface of sphere because charge is centered, we can ignore dot product (cos0˚ = 1)

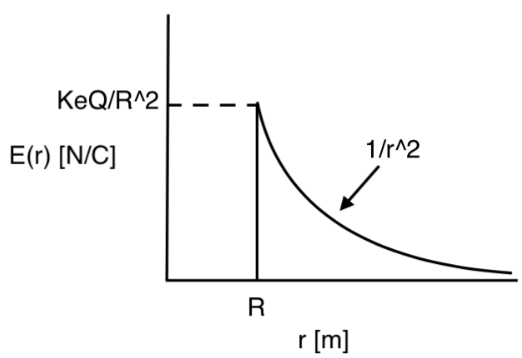
$\oint EdA = \frac{Q}{\varepsilon_o}$

- E is constant because charge is centered

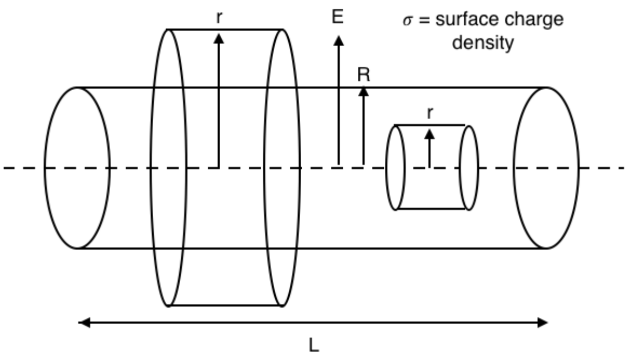
$E\oint dA = \frac{Q}{\varepsilon_o}$

$E * 4\pi r^2 = \frac{Q}{\varepsilon_o}$

E = $\frac{K_e Q}{r^2}$ r̂ when r > R [proves Coulomb's Law, makes sense because sphere is essentially a point charge]



6. Applying Gauss' Law for a conducting metal rod of infinite length



$\sigma = \frac{Q}{A} = \frac{dq}{dA}$ $q_{enc} = \sigma dA = \sigma A$

A = 2πRL

dA = cross-section of gaussian surface

Note: Cylinder caps don't experience electric flux

r < R (inside metal rod)

$\oint E \bullet dA = \frac{q_{enc}}{\varepsilon_o}$

- Since we are inside of the metal rod and all the charge is located on the surface, there is no q_{enc} or $q_{enc} = 0$

E(r) = 0 when r < R

r > R (outside metal rod)

$\oint E \bullet dA = \frac{q_{enc}}{\varepsilon_o}$

- Since E always parallel to dA at surface of rod because charge is centered, we can ignore dot product (cos0° = 1)

$\oint EdA = \frac{\sigma A}{\varepsilon_o}$

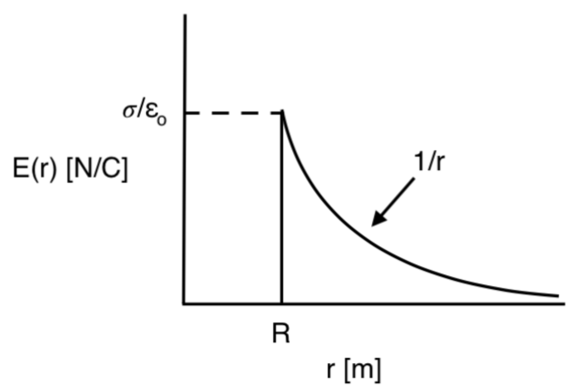
- E is constant because charge is centered

$E \oint dA = \frac{\sigma A}{\varepsilon_o}$

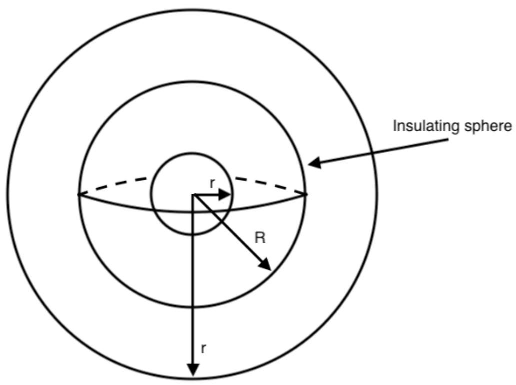
$E * 2\pi rL = \frac{\sigma(2\pi RL)}{\varepsilon_o}$

$E * r = \frac{\sigma R}{\varepsilon_o}$

$E = \frac{\sigma R}{r\varepsilon_o}\hat{r}$ when r > R



7. Applying Gauss' Law for a uniformly distributed charged non-conducting (insulating) sphere (inside and outside)



$\rho = \frac{Q}{V} = \frac{dq}{dV}$ $\quad q_{enc} = \rho dV = \rho V$

$V = \frac{4}{3}\pi r^3$ (r < R / inside)

$V = \frac{4}{3}\pi R^3$ (r > R / outside)

ρ = volumetric charge density

V = volume of gaussian sphere

Note: Volume is different when inside and outside because it is volume of the enclosed charge

- when inside sphere, it depends on the radius (r)
- when outside sphere, it maxes out at radius (R)

r < R (inside insulating sphere)

$\oint E \bullet dA = \frac{q_{enc}}{\varepsilon_o}$

- Since E always parallel to dA at surface of sphere because charge is centered, we can ignore dot product (cos0° = 1)

$\oint E dA = \frac{\rho V}{\varepsilon_o}$

- E is constant because charge is centered

$E \oint dA = \frac{\rho V}{\varepsilon_o}$

$E * 4\pi r^2 = \frac{\rho(4/3\,\pi r^3)}{\varepsilon_o}$

$E = \frac{\rho r}{3\varepsilon_o}\hat{r}$ when r < R

r > R (outside insulating sphere)

$\oint E \bullet dA = \frac{q_{enc}}{\varepsilon_o}$

- Since E always parallel to dA at surface of sphere because charge is centered, we can ignore dot product (cos0° = 1)

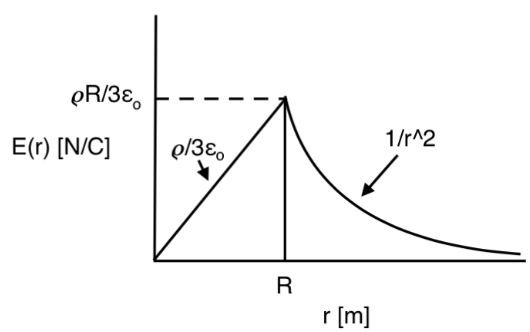
$\oint E dA = \frac{\rho V}{\varepsilon_o}$

- E is constant because charge is centered

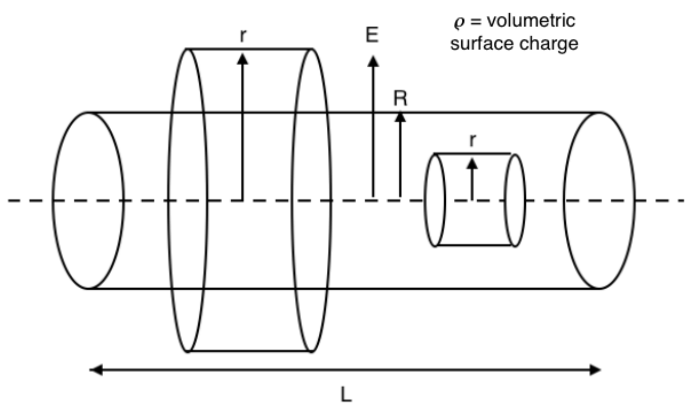
$E \oint dA = \frac{\rho V}{\varepsilon_o}$

$E * 4\pi r^2 = \frac{\rho(4/3\,\pi R^3)}{\varepsilon_o}$

$E = \frac{\rho R^3}{3\varepsilon_o r^2}\hat{r}$ when r > R



8. Applying Gauss' Law for a infinitely long non-conducting (insulating) uniformly distributed charged rod



$\varrho = \frac{Q}{V} = \frac{dq}{dV}$ $q_{enc} = \varrho dV = \varrho V$

$V = \pi r^2 L$ (r < R / inside)

$V = \pi R^2 L$ (r > R / outside)

ϱ = volumetric charge density

V = volume of gaussian cylinder

Note: Cylinder caps don't experience electric flux

Note: Volume is different when inside and outside because it is volume of the enclosed charge

- when inside cylinder, it depends on the radius (r)
- when outside cylinder, it maxes out at radius (R)

r < R (inside insulating rod)

$\oint E \bullet dA = \frac{q_{enc}}{\varepsilon_o}$

- Since E always parallel to dA at surface of rod because charge is centered, we can ignore dot product (cos0˚ = 1)

$\oint EdA = \frac{\varrho V}{\varepsilon_o}$

- E is constant because charge is centered

$E \oint dA = \frac{\varrho V}{\varepsilon_o}$

$E * 2\pi r L = \frac{\varrho(\pi r^2 L)}{\varepsilon_o}$

$E = \frac{\varrho r}{2\varepsilon_o}\hat{r}$ when r < R

r > R (outside insulating rod)

$\oint E \bullet dA = \frac{q_{enc}}{\varepsilon_o}$

- Since E always parallel to dA at surface of rod because charge is centered, we can ignore dot product (cos0˚ = 1)

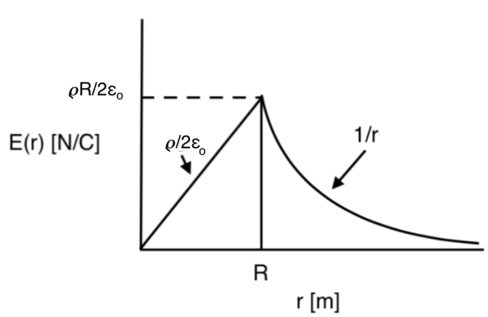
$\oint EdA = \frac{\varrho V}{\varepsilon_o}$

- E is constant because charge is centered

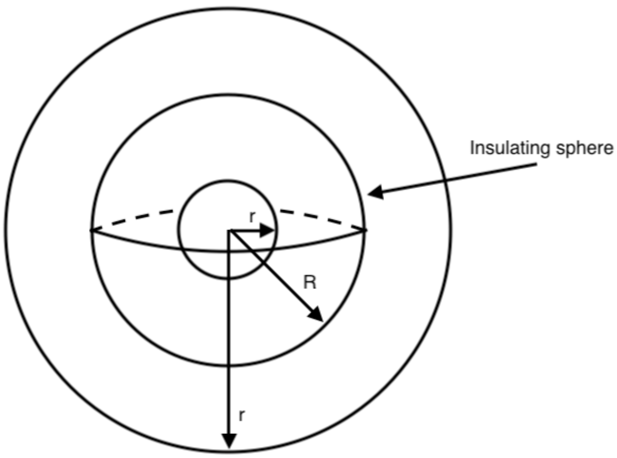
$E \oint dA = \frac{\varrho V}{\varepsilon_o}$

$E * 2\pi r L = \frac{\varrho(\pi R^2 L)}{\varepsilon_o}$

$E = \frac{\varrho R^2}{2\varepsilon_o r}\hat{r}$ when r > R



9. Applying Gauss' Law for a non-conducting (insulating) non-uniformly distributed charged sphere



$\varrho(r) = br$

$\varrho(r)$ = volumetric charge density as a function of radius (r)

V = volume of gaussian cylinder

Note: since q_{enc} changes with radius due to variable charge density, q_{enc} can be calculated with $\int_0^r Density\ *\ SA\ of\ gaussian\ (use\ r)$

$q_{enc} = \int_0^r \varrho(r)\ *\ 4\pi r^2\,dr\ = \int_0^r br\ *\ 4\pi r^2\,dr$

$q_{enc}(r) = \pi b r^4$ (inside)

$q_{enc}(R) = \pi b R^4$ (outside)

r < R (inside insulating sphere)

$\oint E \bullet dA = \frac{q_{enc}}{\varepsilon_o}$

- Since E always parallel to dA at surface of rod because charge is centered, we can ignore dot product (cos0° = 1)

$\oint E dA = \frac{q_{enc}}{\varepsilon_o}$

- E is constant because charge is centered

$E \oint dA = \frac{q_{enc}}{\varepsilon_o}$

$E\ *\ 4\pi r^2 = \frac{\pi b r^4}{\varepsilon_o}$

$E = \frac{br^2}{4\varepsilon_o}\,\hat{r}$ when r < R

r > R (outside insulating sphere)

$\oint E \bullet dA = \frac{q_{enc}}{\varepsilon_o}$

- Since E always parallel to dA at surface of rod because charge is centered, we can ignore dot product (cos0° = 1)

$\oint E dA = \frac{q_{enc}}{\varepsilon_o}$

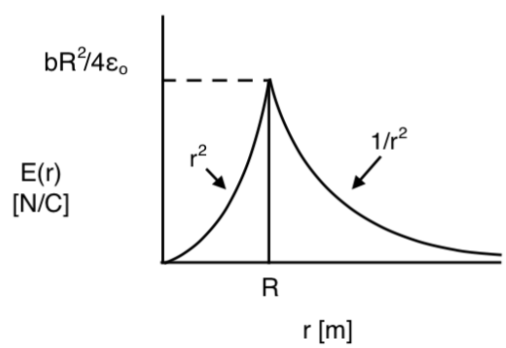
- E is constant because charge is centered

$E \oint dA = \frac{q_{enc}}{\varepsilon_o}$

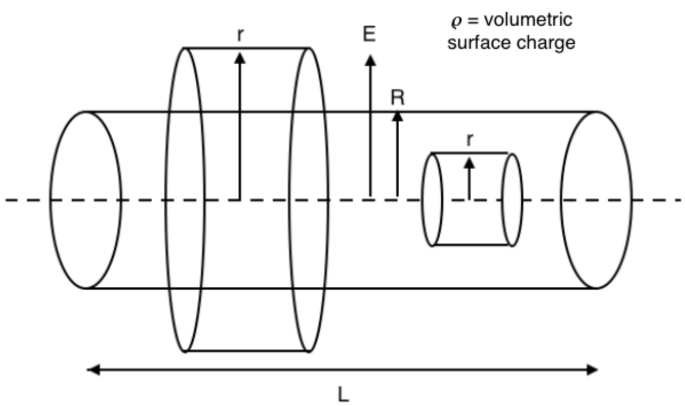
$E\ *\ 4\pi r^2 = \frac{\pi b R^4}{\varepsilon_o}$

$E = \frac{bR^4}{4\varepsilon_o}\,\hat{r}$ when r > R

$\varrho(r)$	E(r) inside varies by
$\varrho(r) = C$	r
$\varrho(r) = br$	r^2
$\varrho(r) = br^2$	r^3
$\varrho(r) = br^3$	r^4



10. Applying Gauss' Law for a non-conducting (insulating) non-uniformly distributed charged rod



$\varrho(r) = ar + b$

$\varrho(r)$ = volumetric charge density as a function of radius (r)

V = volume of gaussian cylinder

Note: since q_{enc} changes with radius due to variable charge density, q_{enc} can be calculated with

$$\int_0^r Density \;*\; SA\ of\ gaussian\ (use\ r)$$

$$q_{enc} = \int_0^r \varrho(r) \;*\; 2\pi rL\, dr$$

$$q_{enc} = \int_0^r (a + br) \;*\; 2\pi rL\, dr$$

$$q_{enc}(r) = 2\pi L\left(\frac{ar^3}{3} + \frac{br^2}{2}\right) \text{ (inside)}$$

$$q_{enc}(R) = 2\pi L\left(\frac{aR^3}{3} + \frac{bR^2}{2}\right) \text{ (outside)}$$

r < R (inside insulating sphere)

$$\oint E \bullet dA = \frac{q_{enc}}{\varepsilon_o}$$

- Since E always parallel to dA at surface of rod because charge is centered, we can ignore dot product (cos0° = 1)

$$\oint E dA = \frac{q_{enc}}{\varepsilon_o}$$

- E is constant because charge is centered

$$E \oint dA = \frac{q_{enc}}{\varepsilon_o}$$

$$E \;*\; 2\pi rL = \frac{2\pi L\left(\frac{ar^3}{3} + \frac{br^2}{2}\right)}{\varepsilon_o}$$

$$E = \frac{ar^2}{3\varepsilon_o} + \frac{br}{2\varepsilon_o}\ \hat{r} \text{ when } r < R$$

r > R (outside insulating sphere)

$$\oint E \bullet dA = \frac{q_{enc}}{\varepsilon_o}$$

- Since E always parallel to dA at surface of rod because charge is centered, we can ignore dot product (cos0° = 1)

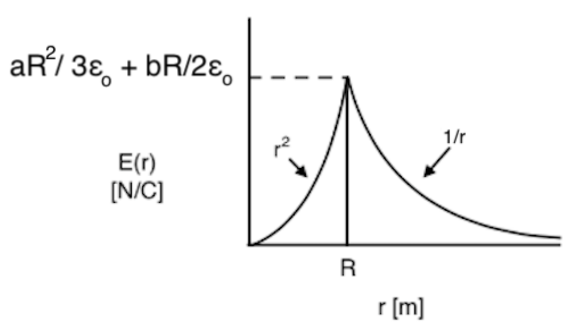
$$\oint E dA = \frac{q_{enc}}{\varepsilon_o}$$

- E is constant because charge is centered

$$E \oint dA = \frac{q_{enc}}{\varepsilon_o}$$

$$E \;*\; 2\pi rL = \frac{2\pi L\left(\frac{aR^3}{3} + \frac{bR^2}{2}\right)}{\varepsilon_o}$$

$$E = \frac{aR^3}{3\varepsilon_o r} + \frac{bR^2}{2\varepsilon_o r}\ \hat{r} \text{ when } r > R$$



TIPS ON HOW TO SOLVE PROBLEMS:

1. IMPORTANT EQUATIONS/CONSTANTS
    a. $\Phi_E = E * A = |E||A|\cos\theta$ [Electric flux]
    b. $\Phi_E = \oint E \bullet dA = \frac{q_{enclosed}}{\varepsilon_o}$ [Gauss' Law]
    c. $E = \frac{F_e}{q}$
    d. $F_e = Eq = ma$
    e. $a = \frac{Eq}{m}$ [For a particle in electric field problems, m = particle mass]
    f. Proton Mass: 1.67×10^{-27} kg
    g. Electron Mass: 9.11×10^{-31} kg
2. Treat particles going through an electric field as projectile motion problems (gravity is just electric force)
    a. Find acceleration using F = ma = Eq then use kinematics
3. Use $\oint E \bullet dA$ part of Gauss' Law when solving for E-field
4. Problem-Solving Steps for Gauss' Law:
    - Right side:
        a. Figure out the relationship between density ($\lambda/\sigma/\varrho$) and $q_{enclosed}$
        b. Ensure the right variables are substituted to match the space of the enclosed charge (similar to Left Side: Step 6 and examples in case study 6 and 7)
        c. When solving for q_{enc}, make sure you substitute A (surface area) or V (volume) with the right variable (r or R) to match the actual enclosed charge
            i. Use r when inside object and R when outside object
        d. Substitute for $q_{enclosed}$ to solve right side of Gauss' Law
    - Left side:
        a. Start with Gauss' Law general equation
        b. Split integrals to all sides of the gaussian surface (top/bottom/sides)
        e. Cancel out surfaces with no electric flux and explain why (because $E \perp A$)
        f. Remove dot product operator and explain why (because E || A)
        g. Move E out of integral and explain why (E is constant because of centered charge or infinite distribution)
        h. Substitute for $\oint$ dA (surface area for GAUSSIAN SURFACE)
        i. Solve for E with algebra
5. Solving q_{enc} tips for Gauss' Law:
    a. If inside metal or conducting object
        i. E = 0
    b. If object conducting (charge on surface) and charge density constant
        i. $q_{enc} = \sigma A$
        ii. Might have to substitute for σ (σ = Q/Total Area)
    c. If object insulating (charge inside) and charge density constant
        i. $q_{enc} = \varrho V$
        ii. Might have to substitute for ϱ (ϱ = Q/Total Volume)

d. If object insulating and charge density changing

i. $q_{enc} = \int_0^r \varrho(r)\ SA\ dr$

ii. SA will always use small r, not big R

UNIT 3: Electrostatic Potential

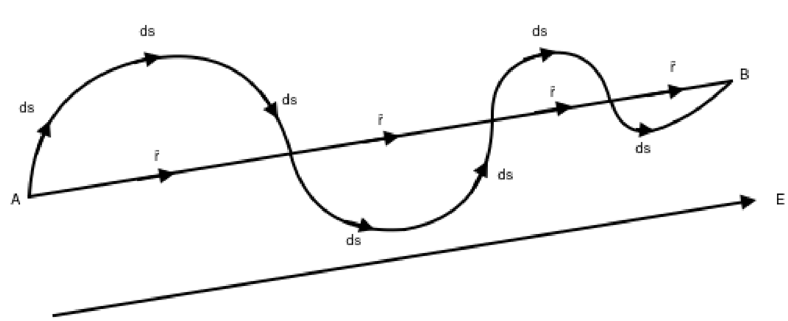
Fundamentals of Unit 3 Physics

Electrostatic Potential (V) [J/C or Volts] : scalar field that is equal to the electrostatic potential energy per unit of charge relative to some point where it is defined to be zero (usually infinity), similarly to gravitational field

- $V = \frac{U_e}{q_o} = \frac{K_e q}{r}$ (specific point in space or with point charges)
- $\Delta V = \frac{\Delta U_e}{q_o} = \frac{W}{q_o} = -\int_A^B E \bullet ds$ (if E is a function of distance)
- U_e = electrostatic potential energy
- ds = differential steps taken from point A to B
- Moving in direction of E-field will lose electrostatic potential and gain kinetic
    - Moving a test charge away from a positive charge
    - Moving a test charge towards a negative charge (similar to Earth's gravity)
- If the charge creating an E-field is positive, then potential should be set to 0 at infinity
- If the charge creating an E-field is negative, then potential should be set to 0 at r=0 (center of the negative charge creating the E-field), similar to gravitational potential
- E-field varies by $1/r^2$ while E-potential varies by 1/r

r̂ ● ds = dr : r̂ ● ds is the radial component of ds which is the same as dr. As you move from point A to point B, how much are you moving radially or along a field line?

- $\hat{r} \bullet ds = |r||ds|\cos\theta$
    - θ is angle between r and ds
- $\hat{r} \bullet ds = dr$
- $\hat{i} \bullet ds = dx$
- $\hat{j} \bullet ds = dy$
- $\check{k} \bullet ds = dz$
- This concept is mainly used for $\Delta V = -\int_{r_A}^{r_B} E \bullet ds$ when E is substituted to integrate in dr domain



FTC - Fundamental Theorem of Calculus : the integral of a function's derivative is the function itself

- $\Delta V = -\int E \bullet dx$ (1-D)
- $E = -\frac{dV}{dx}$ (1-D)

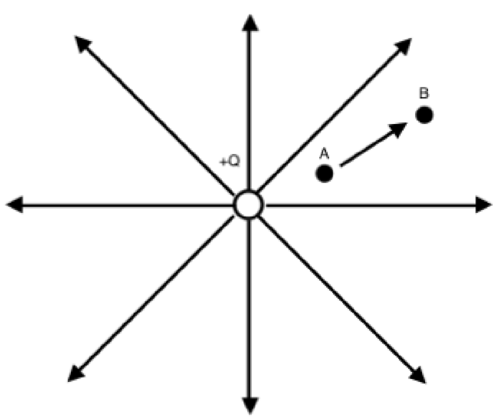
Del Operator (∇) : operator that takes the partial derivative of a multi-dimensional vector, just a symbol to represent a process or procedure essentially. Partial derivative is taking the derivative and treating other variables as constants.

- $\nabla = \frac{\partial}{\partial x}\hat{\imath} + \frac{\partial}{\partial x}\hat{\jmath} + \frac{\partial}{\partial x}\check{k}$
- $E = -\nabla \cdot V = \frac{\partial V}{\partial x}\hat{\imath} + \frac{\partial V}{\partial x}\hat{\jmath} + \frac{\partial V}{\partial x}\check{k}$
- Ex: $-\frac{\partial}{\partial x}(K_e q x^2 y^4 z^2)\,\hat{\imath} = -K_e q(2x)y^4 z^2\,\hat{\imath}$

Equipotential Line/Surface : a plane perpendicular to the E-field at which all points along that plane have the same electrostatic potential energy, plane isn't necessarily straight and uniform, it can curve with the E-field

- Ex: Infinite horizontal equipotential surfaces above an infinite sheet of charge
- Ex: Infinite radial equipotential surfaces around (orbit-like) a point or spherical charge
- Closer equipotential surfaces means a greater potential gradient so E-field is greater because of the formula: $E = -\frac{dV}{dx}$

Potential near a point charge



Going from point A to B **loses** electrostatic potential energy and **gains** kinetic energy as it **moves in the same direction as the E-field**

$$\Delta V_{A\to B} = V_B - V_A = -\int_{r_A}^{r_B} E \bullet ds$$

$$\Delta V_{A\to B} = -\int_{r_A}^{r_B} E \bullet ds$$ [substitute E for $\frac{K_e q}{r^2}\hat{r}$]

$$\Delta V_{A\to B} = -\int_{r_A}^{r_B} \frac{K_e q}{r^2}\hat{r} \bullet ds$$ [replace $\hat{r} \bullet ds$ with dr]

$$\Delta V_{A\to B} = -\int_{r_A}^{r_B} \frac{K_e q}{r^2}\, dr$$ [integrate]

$$\Delta V_{A\to B} = K_e q\left(\frac{1}{r_B} - \frac{1}{r_A}\right)$$ [apply limits]

Absolute potential near a point charge

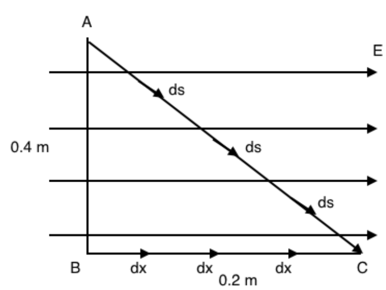
$\Delta V_{A\rightarrow B} = V_B - V_A$

$V_B - V_A = K_e Q[\frac{1}{r_B} - \frac{1}{r_A}]$ [proven from "Potential near a point charge" section above]

- $V_A = 0$ by definition
- $\frac{1}{r_A} = \frac{1}{\infty} = 0$ by math

$V_B = K_e Q[\frac{1}{r_B}] = \frac{K_e Q}{r}$ [proving $V = \frac{K_e Q}{r}$]

Potential in uniform and varying E-fields



Note: $\Delta V_{A\rightarrow B} = 0$ as they are on the same equipotential line

Note: This is the long solution showing both $\Delta V_{A\rightarrow B} + \Delta V_{B\rightarrow C}$

E = 50 N/C î

$\Delta V_{A\rightarrow C} = \Delta V_{A\rightarrow B} + \Delta V_{B\rightarrow C} = \Delta V_{B\rightarrow C}$

$\Delta V_{A\rightarrow C} = -\int_A^B E \bullet ds - \int_B^C E \bullet ds$ [substituting for ΔV]

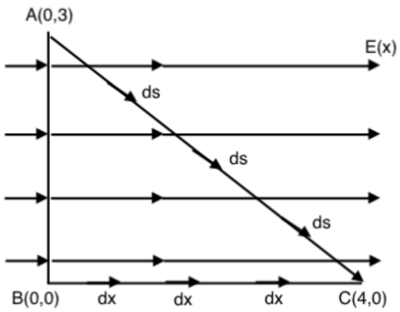
$\Delta V_{A\rightarrow C} = -\int_A^B 50\,\hat{\imath} \bullet ds - \int_B^C 50\,\hat{\imath} \bullet ds$ [substituting for E]

$\Delta V_{A\rightarrow C} = -\int_A^B 50\,dx - \int_B^C 50\,dx$ [$\hat{\imath} \bullet ds = dx$]

$\Delta V_{A\rightarrow C} = -\ 50x|_A^B - 50x|_B^C$ [integrate]

$\Delta V_{A\rightarrow C} = -\ 50\hat{\imath}(0\hat{\imath}) - 50\hat{\imath}(0.2\hat{\imath})$ [apply limits for $x_{A\rightarrow B}/x_{B\rightarrow C}$]

$\Delta V_{A\rightarrow C}$ = -10 J/C



Note: $\Delta V_{A\to B} = 0$ as they are on the same equipotential line

Note: This is the short solution showing only $\Delta V_{B\to C}$

E = 4x N/C î

$\Delta V_{A\to C} = \Delta V_{A\to B} + \Delta V_{B\to C} = \Delta V_{B\to C}$

$\Delta V_{A\to C} = -\int_B^C E \bullet ds$ [substituting for ΔV]

$\Delta V_{A\to C} = -\int_0^4 4x\,\hat{\imath} \bullet ds$ [substituting for E]

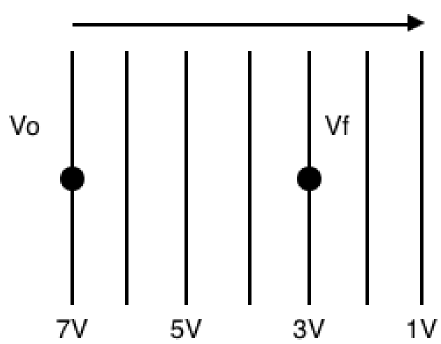
$\Delta V_{A\to C} = -\int_0^4 4x\,dx$ [$\hat{\imath} \bullet ds = dx$]

$\Delta V_{A\to C} = -\ 2x^2\big|_0^4\ dx$ [integrate]

$\Delta V_{A\to C} = -\ 2(4)^2 - 0$ [apply limits]

$\Delta V_{A\to C} = -32$ J/C

Applying Conservation of Mechanical Energy to E-field/potential problems



Proton has $V_o = 0$ at 7V

Remember: $U_e = Vq_o$

$$\Delta E_{mech} = 0$$

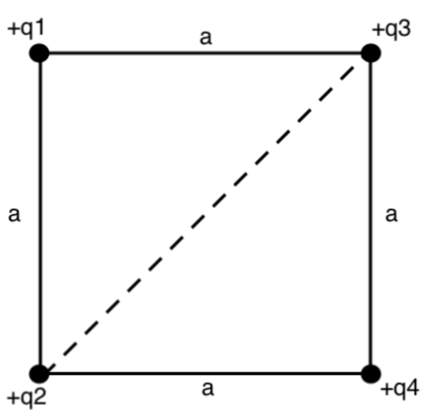
$$U_{Eo} + K_o = U_{Ef} + K_f$$

$$V_o q = V_f q + \tfrac{1}{2} m v_f^2$$

$$v_f = \sqrt{\frac{2q(\Delta v)}{m}}$$

Note: Electron would move left, opposite of the E-field

Binding energy of an assembly of charges



Remember: $U_e = Vq_o$ $V = \frac{K_e q}{r}$

Work/Energy required to bring charge from infinity:

$\Delta U_1 = 0$

$\Delta U_2 = \frac{K_e q_1}{a} q_2$

$\Delta U_3 = \frac{K_e q_1}{a} q_3 + \frac{K_e q_2}{\sqrt{2}a} q_3$

$\Delta U_4 = \frac{K_e q_1}{\sqrt{2}a} q_4 + \frac{K_e q_2}{a} q_4 + \frac{K_e q_3}{a} q_4$

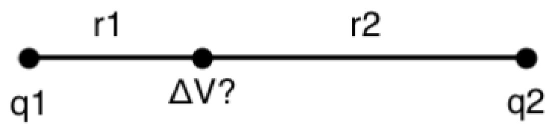
$q_1 = q_2 = q_3 = q_4 = q$

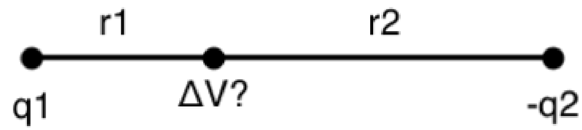
$$\Delta U_{total} = W_{total} = 4\frac{K_e q^2}{a} + 2\frac{K_e q^2}{\sqrt{2}a}$$

Problem-Solving: List of Potential Problems

1. Potential due to discrete (point) charges
2. Potential at the center of charged ring
3. Potential due to uniformly distributed ring of charge, off-center along center axis
4. Potential due to uniformly distributed charged arc
5. Potential due to an electric dipole (axis of a dipole and perpendicular bisector)
6. Potential due to uniformly charged rod (axis of a rod and perpendicular bisector)
7. Potential due to uniformly distributed disk of charge, off-center along center axis
8. Potential at the center of a uniformly distributed charged insulating sphere
9. Potential inside a uniformly distributed charged insulating sphere

1. Potential due to discrete (point) charges





$V_1 = \frac{K_e q_1}{r_1}$ $V_2 = \frac{K_e q_2}{r_2}$

$\Delta V = V_1 + V_2 = \frac{K_e q_1}{r_1} + \frac{K_e q_2}{r_2}$

ΔV_{max} at the center between q_1 and q_2

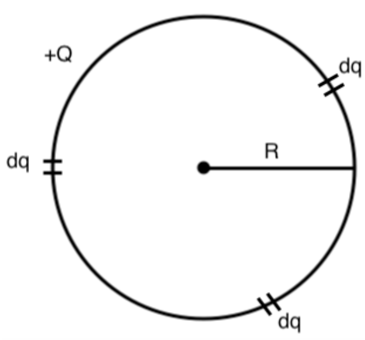
$V_1 = \frac{K_e q_1}{r_1}$ $V_2 = \frac{K_e(-q_2)}{r_2}$

$\Delta V = V_1 + V_2 = \frac{K_e q_1}{r_1} - \frac{K_e q_2}{r_2}$

ΔV_{min} at the center between q_1 and $-q_2$

Note: Only negative charges will decrease the potential (ΔV), never the distance from a charge

2.Potential due at the center of charged ring



$V_c = \frac{K_e q}{r}$ [general eq.]

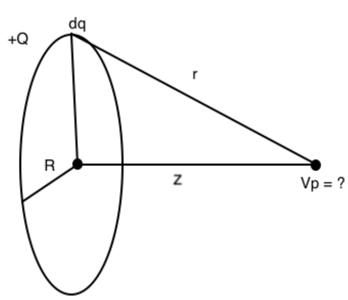
$dV_c = \frac{K_e dq}{R}$ [turn into differential and replace r]

$V_c = \int \frac{K_e dq}{R}$ [integrate dq]

$V_c = \frac{K_e}{R} \int dq$ [move constants out]

$V_c = \frac{K_e Q}{R}$

3. Potential due to uniformly distributed ring of charge, off-center along center axis



$V = \frac{K_e q}{r}$ [general eq.]

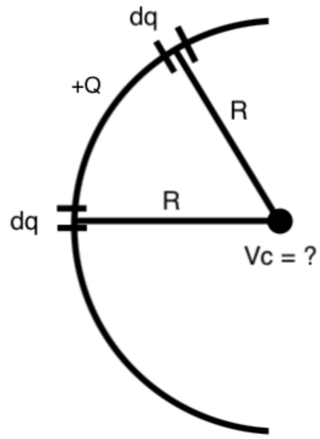
$dV = \frac{K_e dq}{\sqrt{R^2+z^2}}$ [turn into differential and replace r]

$V = \int \frac{K_e dq}{\sqrt{R^2+z^2}}$ [integrate dq]

$V_c = \frac{K_e}{\sqrt{R^2+z^2}} \int dq$ [move constants out]

$V_c = \frac{K_e Q}{\sqrt{R^2+z^2}}$

4. Potential due to uniformly distributed charged arc



$V_c = \frac{K_e q}{r}$ [general eq.]

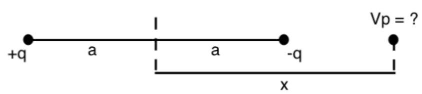
$dV_c = \frac{K_e dq}{R}$ [turn into differential and replace r]

$V_c = \int \frac{K_e dq}{R}$ [integrate dq]

$V_c = \frac{K_e}{R}\int dq$ [move constants out]

$V_c = \frac{K_e Q}{R}$

5. Potential due to an electric dipole (axis of a dipole and perpendicular bisector)



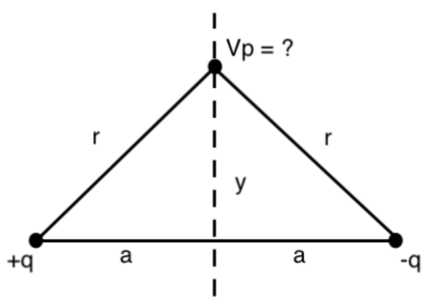
$V_p = V_{p+} + V_{p-}$ [general eq.]

$V_p = \frac{K_e q}{r} - \frac{K_e q}{r}$ [substitute for V]

$V_p = \frac{K_e q}{x+a} - \frac{K_e q}{x-a}$ [substitute for r]

$V_p = K_e q(\frac{1}{x+a} - \frac{1}{x-a})$ [factor out $K_e q$]

$V_p = \frac{-2aK_e q}{x^2-a^2}$



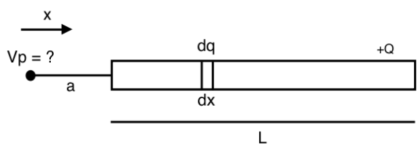
$V_p = V_{p+} + V_{p-}$ [general eq.]

$V_p = \frac{K_e q}{r} - \frac{K_e q}{r}$ [substitute for V]

$V_p = \frac{K_e q}{\sqrt{x^2+a^2}} - \frac{K_e q}{\sqrt{x^2+a^2}}$ [substitute for r]

$V_p = 0$ (potential = 0 at perpendicular bisector of a dipole)

6. Potential due to uniformly charged rod (axis of a rod and perpendicular bisector)



$\lambda = \frac{Q}{L} = \frac{dq}{dx}$ $\quad$ $dq = \lambda dx$

$V = \frac{K_e q}{r}$ [general eq.]

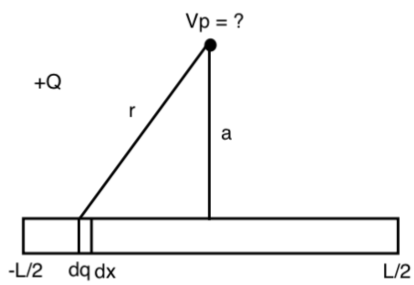
$dV = \frac{K_e dq}{x}$ [turn into differential and replace r]

$V = \int_a^{a+L} \frac{K_e \lambda dx}{x}$ [integrate dx, apply limits, replace dq]

$V = K_e \lambda \int_a^{a+L} \frac{dx}{x}$ [move constants out]

$V = K_e \lambda ln|x|_a^{a+L}$ [integrate dx and evaluate]

$V = K_e \lambda ln[\frac{a+L}{a}]$



$\lambda = \frac{Q}{L} = \frac{dq}{dx}$ $\quad$ $dq = \lambda dx$

$V = \frac{K_e q}{r}$ [general eq.]

$dV = \frac{K_e dq}{\sqrt{x^2+a^2}}$ [turn into differential and replace r]

$V = \int_{-L/2}^{L/2} \frac{K_e \lambda dx}{\sqrt{x^2+a^2}}$ [integrate dx, apply limits, replace dq]

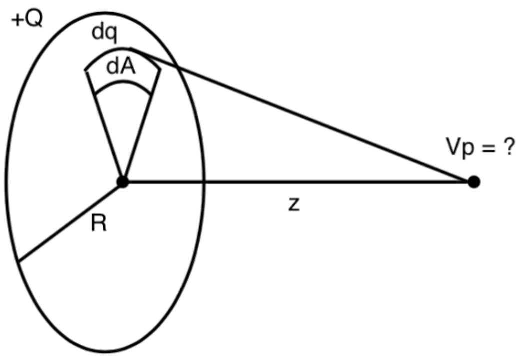
$V = K_e \lambda \int_{-L/2}^{L/2} \frac{dx}{\sqrt{x^2+a^2}}$ [move constants out]

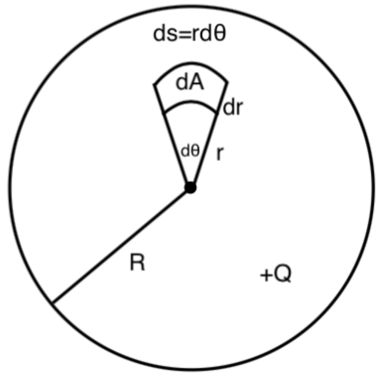
$V = K_e \lambda ln|x + \sqrt{x^2 + a^2}|_{-L/2}^{L/2}$ [integrate dx and evaluate]

$V = K_e \lambda ln[\frac{\frac{L}{2}+\sqrt{\frac{L^2}{4}+a^2}}{\frac{L}{2}-\sqrt{\frac{L^2}{4}+a^2}}]$ [simplify]

$V = K_e \lambda ln[\frac{L+\sqrt{L^2+4a^2}}{-L+\sqrt{L^2+4a^2}}]$

7. Potential due to uniformly distributed disk of charge, off-center along center axis





$\sigma = \frac{Q}{\pi R^2} = \frac{dq}{dA} = \frac{dq}{r d\theta dr}$

dq = σrdrdθ

$V = \frac{K_e q}{r}$	[general eq.]
$dV = \frac{K_e dq}{\sqrt{r^2+z^2}}$	[turn into differential and replace r]
$V = \int_0^R \int_0^{2\pi} \frac{K_e \sigma r dr d\theta}{\sqrt{r^2+z^2}} = \int_0^R \frac{K_e \sigma r dr}{\sqrt{r^2+z^2}} \int_0^{2\pi} d\theta$	[integrate dr and dθ, apply limits, replace dq]
$V = \int_0^R \frac{K_e \sigma r dr}{\sqrt{r^2+z^2}} (2\pi)$	[evaluate dθ]
$V = 2\pi K_e \sigma \int_0^R \frac{r dr}{\sqrt{r^2+z^2}}$	[move out constants]
$V = 2\pi K_e \sigma(\sqrt{r^2 + z^2})_0^R$	[integrate dr and evaluate]
$V = 2\pi K_e \sigma(\sqrt{R^2 + z^2} - z)$	

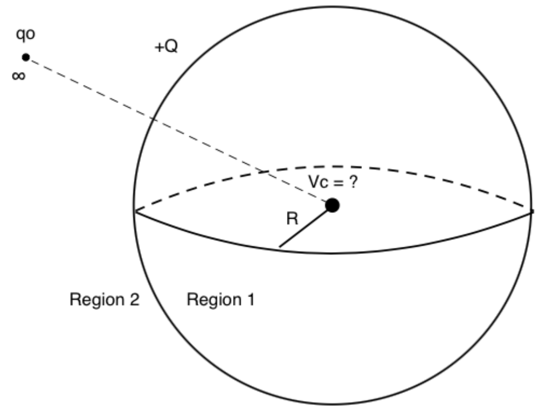
V_c at the center: (z = 0)

Remember: Q = $\sigma\pi R^2$ $\quad \sigma = \frac{Q}{\pi R^2}$

$V_c = 2\pi K_e \sigma(\sqrt{R^2 + z^2} - z)$	[general eq.]
$V_c = 2\pi K_e \sigma R$	[simplify when z = 0]
$V_c = 2\pi K_e (\frac{Q}{\pi R^2}) R$	[substitute for σ]
$V_c = \frac{2K_e Q}{R}$	

Note the difference when compared to $E = \frac{K_e Q}{R}$

8. Potential at the center of a uniformly distributed charged insulating sphere



Using Gauss' Law, E(r) can be found inside and outside the sphere:

Region 1 (inside): $E_1(r) = \frac{K_e Q r}{R^3}\,\hat{r}$ $\quad r < R$

Region 2 (outside): $E_2(r) = \frac{K_e Q}{r^2}\,\hat{r}$ $\quad r > R$

Method to find potential: bringing a test charge from infinity and placing it at the surface of the sphere then to the center (this accounts for the different E-fields)

Note: V_C = potential at the center where r = 0

$\Delta V_{\infty\to C} = V_C - V_\infty = V_C$ [$V_\infty = 0$ by definition]

$\Delta V_{\infty\to C} = \Delta V_{\infty\to R} + \Delta V_{R\to C} = V_R - V_\infty + V_C - V_R = V_C$ [proves $\Delta V_{\infty\to C} = \Delta V_{\infty\to R} + \Delta V_{R\to C}$]

$\Delta V_{\infty\to C} = -\int_\infty^R E_2 \bullet ds - \int_R^0 E_1 \bullet ds$ [substituting for $\Delta V_{\infty\to R} + \Delta V_{R\to C}$]

$\Delta V_{\infty\to C} = -\int_\infty^R \frac{K_e Q}{r^2}\,\hat{r} \bullet ds - \int_R^0 \frac{K_e Q r}{R^3}\,\hat{r} \bullet ds$ [substituting for E]

$\Delta V_{\infty\to C} = -\int_\infty^R \frac{K_e Q}{r^2}\,dr - \int_R^0 \frac{K_e Q r}{R^3}\,dr$ [$\hat{r} \bullet ds = dr$]

$\Delta V_{\infty\to C} = -K_e Q \int_\infty^R \frac{1}{r^2}\,dr - \frac{K_e Q}{R^3}\int_R^0 r\,dr$ [move out constants]

$\Delta V_{\infty\to C} = -K_e Q\left(\frac{-1}{r}\right)_\infty^R - \frac{K_e Q}{R^3}\left(\frac{r^2}{2}\right)_R^0$ [integrate dr]

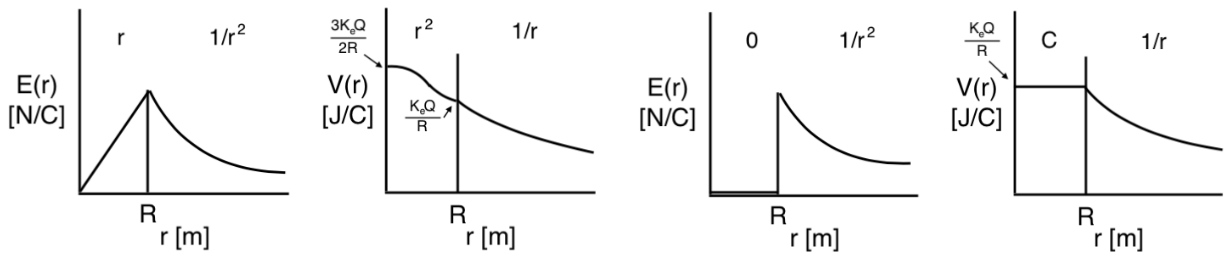
$\Delta V_{\infty\to C} = -K_e Q\left(\frac{-1}{R} + \frac{1}{\infty}\right) - \frac{K_e Q}{R^3}\left(0 - \frac{R^2}{2}\right)$ [evaluate limits]

$\Delta V_{\infty\to C} = \frac{K_e Q}{R} + \frac{K_e Q}{2R}$ [simplify]

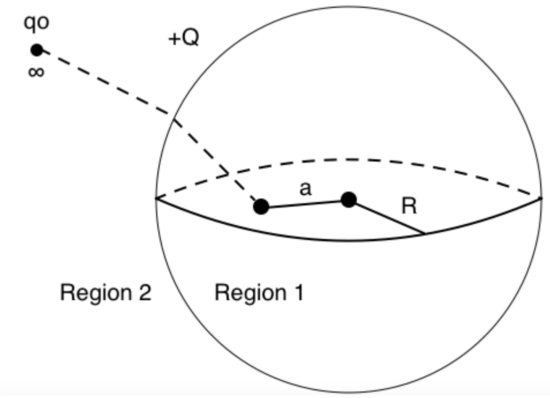
$\Delta V_{\infty\to C} = \frac{3K_e Q}{2R}$

Graphing E-field and Potential for:

Insulating Sphere (corresponds with above) Conducting Sphere



9. Potential inside a uniformly distributed charged insulating sphere



Using Gauss' Law, E(r) can be found inside and outside the sphere:

Region 1 (inside): $E_1(r) = \frac{K_e Q r}{R^3}\,\hat{r}$ $\quad r < R$

Region 2 (outside): $E_2(r) = \frac{K_e Q}{r^2}\,\hat{r}$ $\quad r > R$

Method to find potential: bringing a test charge from infinity and placing it at the surface of the sphere then to a point "a" distance away from the center (this accounts for the different E-fields)

$\Delta V_{\infty\to a} = V_a - V_\infty = V_a$ [$V_\infty = 0$ by definition]

$\Delta V_{\infty\to a} = \Delta V_{\infty\to R} + \Delta V_{R\to a} = V_R - V_\infty + V_a - V_R = V_a$ [proves $\Delta V_{\infty\to a} = \Delta V_{\infty\to R} + \Delta V_{R\to a}$]

$\Delta V_{\infty\to a} = -\int_\infty^R E_2 \bullet ds - \int_R^a E_1 \bullet ds$ [substituting for $\Delta V_{\infty\to R} + \Delta V_{R\to C}$]

$\Delta V_{\infty\to a} = -\int_\infty^R \frac{K_e Q}{r^2}\,\hat{r} \bullet ds - \int_R^a \frac{K_e Q r}{R^3}\,\hat{r} \bullet ds$ [substituting for E]

$\Delta V_{\infty\to a} = -\int_\infty^R \frac{K_e Q}{r^2}\,dr - \int_R^a \frac{K_e Q r}{R^3}\,dr$ [$\hat{r} \bullet ds = dr$]

$\Delta V_{\infty\to a} = -K_e Q \int_\infty^R \frac{1}{r^2}\,dr - \frac{K_e Q}{R^3}\int_R^a r\,dr$ [move out constants]

$\Delta V_{\infty\to a} = -K_e Q\left(\frac{-1}{r}\right)_\infty^R - \frac{K_e Q}{R^3}\left(\frac{r^2}{2}\right)_R^a$ [integrate dr]

$\Delta V_{\infty\to a} = -K_e Q\left(\frac{-1}{R} + \frac{1}{\infty}\right) - \frac{K_e Q}{R^3}\left(\frac{a^2}{2} - \frac{R^2}{2}\right)$ [evaluate limits]

$\Delta V_{\infty\to a} = \frac{K_e Q}{R} - \frac{K_e Q a^2}{2R^3} + \frac{K_e Q R^2}{2R^3}$ [simplify]

$\Delta V_{\infty\to a} = \frac{K_e Q}{R} - \frac{K_e Q a^2}{2R^3} + \frac{K_e Q}{2R}$ [simplify]

$\Delta V_{\infty\to a} = \frac{3K_e Q}{2R} - \frac{K_e Q a^2}{2R^3}$ [simplify]

$\Delta V_{\infty\to a} = \frac{K_e Q}{R}\left[\frac{3}{2} - \frac{a^2}{2R^2}\right]$

TIPS ON HOW TO SOLVE PROBLEMS:

1. IMPORTANT EQUATIONS/CONSTANTS
   a. $V = \frac{U_e}{q_o} = \frac{K_e q}{r}$
   b. $\Delta V = \frac{\Delta U_e}{q_o} = \frac{W}{q_o} = -\int_A^B E \bullet ds$
   c. $E = -\nabla \cdot V = \frac{\partial V}{\partial x}\,\hat{i} + \frac{\partial V}{\partial x}\,\hat{j} + \frac{\partial V}{\partial x}\,\check{k}$
2. List of useful integrals: (a = constant)
   a. $\int \frac{dx}{(x^2+a^2)^{1/2}} = ln(x + \sqrt{x^2 + a^2})$ (used in bisector axis of charged rod)
3. Problem-solving steps for potential problems (point charge / 2D objects):
   a. Establish relationships with densities (σ/λ) and/or find out what dq equals
   b. Start with general equation $V = \frac{K_e q}{r}$
   c. Turn general equation into differential and replace r
   d. Substitute for dq from step "a"
   e. Integrate over necessary domains (dr and/or dθ) and move out constants
   f. Apply limits to appropriate domain
   g. Integrate and evaluate the integral
   h. Simplify
4. Problem-solving steps for potential problems (3D objects):
   a. Find out E-fields of each region using Gauss' Law (see unit 2)
   b. Start with general equation $\Delta V = -\int_A^B E \bullet ds$
   c. Split the path from infinity to the desired point and give the general equation to each separated path
      i. Infinity to surface, surface to point
   d. Apply limits to respective separated paths
      i. Infinity to surface: ∞→R, surface to point: R→?
   e. Substitute for E-field found in step "a" for each respective region of the separated paths
      i. Infinity to surface: sub E-field outside, surface to point: sub E-field inside
      ii. Don't forget the $\hat{r}$ that comes with E-field
   f. Replace $\hat{r} \bullet ds$ to dr
   g. Move out constants
   h. Integrate over domain of dr
   i. Simplify
5. The magnitude of potential is MAX at the center between 2 charges of the same polarity (both positive/negative)
6. The magnitude of potential is MINIMUM or 0 between 2 charges of opposite polarity (one positive one negative and vice versa)
7. Moving in the same direction as E-field will lose potential energy
8. When using conservation of mechanical energy in solving potential problems and the initial point is very far away from the final point, the initial point will have 0 potential energy

UNIT 4: Capacitors, Dielectrics, and Conductors

Fundamentals of Unit 4 Physics

Capacitance (F) [C/V or Farads] : amount of charge stored in a capacitor per unit of potential difference

- $C = \frac{Q}{\Delta V}$
- Applying a potential difference will allow the capacitor to hold a charge
- Capacitor is independent of the amount of charge or potential difference, ONLY the geometry and the dielectric material
- Can't be negative
- Problem solving steps:
    - Find E_{gap} with Gauss' Law
    - Find ΔV using E_{gap} and integrate from low to high potential
        - Change E_{gap} into domain of dr if necessary
    - Find C with Q and ΔV

Energy of a Capacitor (J) [Joules] : amount of energy in a capacitor

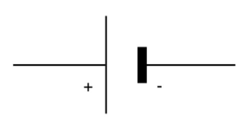
- $W = \frac{1}{2}\frac{Q^2}{C}$
- $W = \frac{1}{2}Q\Delta V$
- $W = \frac{1}{2}C\Delta V^2$

Energy Density (u) [J/V or Joules/Volume] : energy stored in a capacitor per unit of gap volume

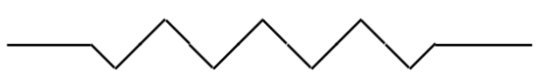
Symbols in Circuit Diagrams :

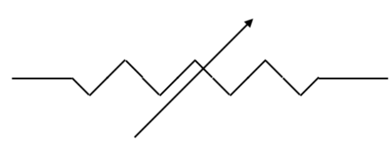
Wire

Battery



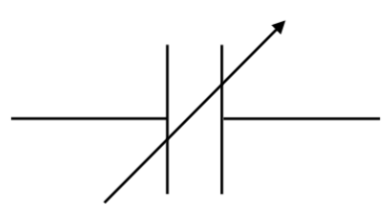
Lamp Resistance Device





(variable)

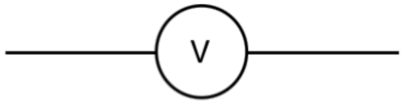
Capacitors

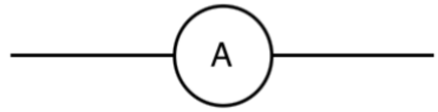


(variable)

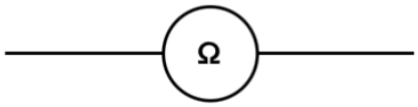
Voltmeter

Ammeter

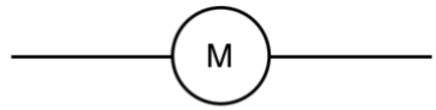




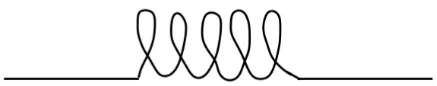
Ohmmeter



Multimeter



Inductor



Switch

Series Connections : capacitors are connected in a series or one after another

- Charges on all capacitors are the same
    - $Q_{total} = Q_1 = Q_2 = Q_3$
- Potential differences / Voltages are different (distributed throughout capacitors)
    - $\Delta V_{total} = \Delta V_1 + \Delta V_2 + \Delta V_3$
- Equivalent Capacitance formula:
    - $C_{eq} = \frac{1}{\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}}$

Parallel Connections : capacitors are connected in parallel or side by side one another

- Charges on all capacitors are different (distributed throughout capacitors)
    - $Q_{total} = Q_1 + Q_2 + Q_3$
- Potential differences / Voltages are the same
    - $\Delta V_{total} = \Delta V_1 = \Delta V_2 = \Delta V_3$
- Equivalent Capacitance formula:
    - $C_{eq} = C_1 + C_2 + C_3$

Dielectric : an insulator that is placed between a capacitor to reduce the potential difference at which the charges are held, increasing the capacitance

- Achieved by polarization of insulator's atoms which creates an E-field in the opposite direction, lowering potential difference

Dielectric Constant (k) : a constant to quantitatively measure the dielectric ability to a material

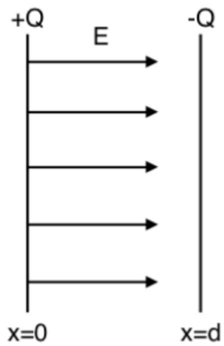
- Can't be negative and lowest is 1

- Dielectric constants of some materials:
    - Vacuum = 1 (base)
    - Air >= 1
    - Plastic = 3
    - Paper = 3.5
    - Teflon = 2.5
    - Rubber = 7
    - Water = 80 (already polarized)
    - Strontium Titanate = 233
- Permittivity of dielectric
    - $\epsilon_k = \epsilon_o k$

Problem-Solving: List of Capacitance Problems

1. Capacitance of a parallel plate capacitor
2. Capacitance of a spherical capacitor
3. Capacitance of a cylindrical capacitor

1. Capacitance of a parallel plate capacitor



$E_{gap} = E_+ + E_- = \frac{\sigma_+}{2\epsilon_o} + \frac{\sigma_-}{2\epsilon_o} = \frac{\sigma}{\epsilon_o} = \frac{Q}{A\epsilon_o}$ [getting E_{gap}]

$\Delta V = -\int_d^0 E_{gap} \cdot ds$ (integrate from low to high potential) [general eq.]

$\Delta V = -E\int_d^0 ds$ [E constant, take out]

$\Delta V = -E(-d) = Ed$ [integrate]

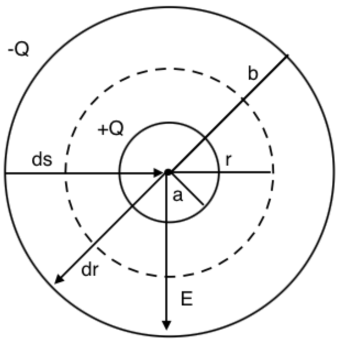
$\Delta V = \frac{Qd}{A\epsilon_o}$ [sub for E_{gap}]

$C_{\parallel} = \frac{Q}{\Delta V}$ [general eq.]

$C_{\parallel} = \frac{QA\epsilon_o}{Qd}$ [sub for ΔV]

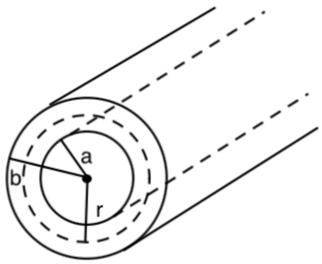
$C_{\parallel} = \frac{A\epsilon_o}{d}$ [simplify]

2. Capacitance of a spherical capacitor



$\oint E_{gap} \cdot dA = \frac{q_{enclosed}}{\epsilon_o}$	[getting E_{gap}]
$E_{gap}(4\pi r^2) = \frac{Q}{\epsilon_o}$	[getting E_{gap}]
$E_{gap} = \frac{Q}{4\pi r^2 \epsilon_o}$	[getting E_{gap}]
$\Delta V = -\int_b^a E_{gap} \cdot ds$	[general eq.]
$E_{gap} \cdot ds = -E_{gap} ds = E_{gap} dr$	[get E in domain of dr]
$\Delta V = -\int_b^a E_{gap} dr$	[substitute $E_{gap} \cdot ds$]
$\Delta V = -\frac{Q}{4\pi\epsilon_o}\int_b^a \frac{1}{r^2} dr$	[integrate]
$\Delta V = -\frac{Q}{4\pi\epsilon_o}\frac{-1}{r}\Big\vert_b^a$	[evaluate limits]
$\Delta V = -\frac{Q}{4\pi\epsilon_o}(\frac{-1}{a} + \frac{1}{b})$	[simplify]
$\Delta V = \frac{Q}{4\pi\epsilon_o}(\frac{b-a}{ab})$	[simplify]
$C_{\parallel} = \frac{Q}{\Delta V}$	[general eq.]
$C_{\parallel} = \frac{Q}{\frac{Q}{4\pi\epsilon_o}(\frac{b-a}{ab})}$	[sub for ΔV]
$C_{\parallel} = 4\pi\epsilon_o(\frac{ab}{b-a})$	

3. Capacitance of a cylindrical capacitor



$\oint E_{gap} \cdot dA = \frac{q_{enclosed}}{\epsilon_o}$ [getting E_{gap}]

$E_{gap}(2\pi rL) = \frac{Q}{\epsilon_o}$ [getting E_{gap}]

$E_{gap} = \frac{Q}{2\pi rL\epsilon_o}$ [getting E_{gap}]

$\Delta V = -\int_b^a E_{gap} \cdot ds$ [general eq.]

$E_{gap} \cdot ds = -E_{gap}ds = E_{gap}dr$ [get E in domain of dr]

$\Delta V = -\int_b^a E_{gap}dr$ [substitute $E_{gap} \cdot ds$]

$\Delta V = -\frac{Q}{2\pi L\epsilon_o}\int_b^a \frac{1}{r}dr$ [integrate]

$\Delta V = -\frac{Q}{2\pi L\epsilon_o} ln|r||_b^a$ [evaluate limits]

$\Delta V = -\frac{Q}{2\pi L\epsilon_o} ln|\frac{a}{b}|$ [simplify]

$C_{\parallel} = \frac{Q}{\Delta V}$ [general eq.]

$C_{\parallel} = \frac{Q}{-\frac{Q}{2\pi L\epsilon_o} ln|\frac{a}{b}|}$ [sub for ΔV]

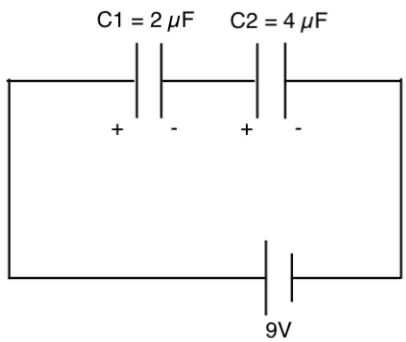
$C_{\parallel} = \frac{2\pi L\epsilon_o}{ln|\frac{b}{a}|}$

Reconnecting Capacitors (Series → Parallel)

Steps in reconnecting capacitors from series to parallel:

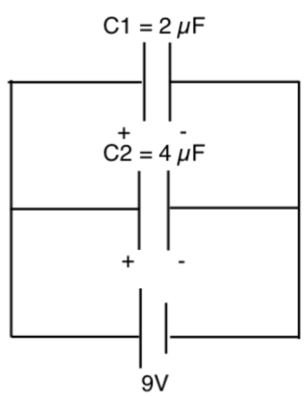
1. Find C_{eq} (series)
2. Find Q_{total} and Q on each capacitor
3. Reconnect capacitors in parallel
4. Find ΔV across each capacitor
5. Find charge on each capacitor

Ex:



1. $C_{eq} = \frac{1}{\frac{1}{C_1}+\frac{1}{C_2}} = 1.33\mu F$
2. $Q = C_{eq}\Delta V = 1.33 * 9 = 12\mu C$

   $Q_{total} = nQ = 2 * 12 = 24\mu C$



3.
4. $\Delta V = \frac{Q_{total}}{C_{eq}\,(parallel)} = \frac{24}{6} = 4V$
5. $Q_1 = C_1\Delta V = 2 * 4 = 8\mu C$

   $Q_2 = C_2\Delta V = 4 * 4 = 16\mu C$

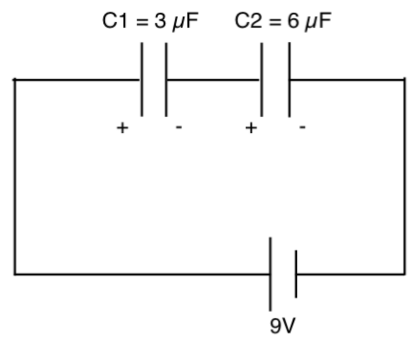
   $Q_{total} = 8 + 16 = 24\mu C$

Reconnecting Capacitors (Parallel → Series)

Steps in reconnecting capacitors from parallel to series:

1. Find C_{eq} (parallel)
2. Find Q_1 and Q_2 to find Q_{total}
3. Reconnect capacitors in series
4. Find charge on each capacitor
5. Find ΔV across each capacitor

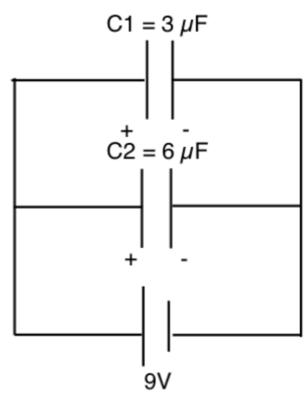
Ex:



1. $C_{eq} = C_1 + C_2 = 9\mu F$
2. $Q_1 = C_1\Delta V = 3 * 9 = 27\mu C$

   $Q_2 = C_2\Delta V = 6 * 9 = 54\mu C$

   $Q_{total} = 27 + 54 = 81\mu C$

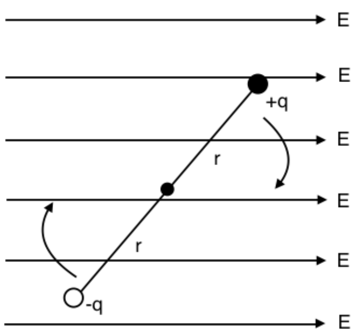


3.
4. $Q = \frac{Q_{total}}{n} = \frac{81}{2} = 40.5\mu C$
5. $\Delta V_1 = \frac{Q}{C_t} = \frac{40.5}{3} = 13.5V$

   $\Delta V_2 = \frac{Q}{C_2} = \frac{40.5}{6} = 6.75V$

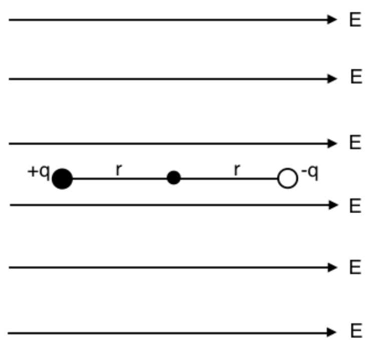
   $\Delta V_{total} = 13.5 + 6.75 = 20.25V$

<u>Electric Dipole in an E-field</u>



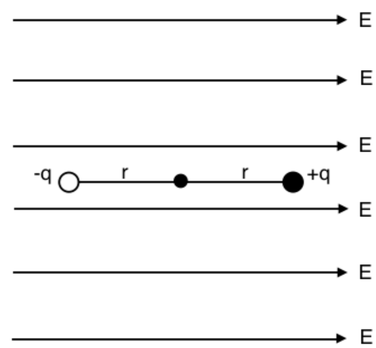
Experiencing Torque
r = moment arm
p = electric dipole moment
p = qr

$\tau = p \times E = pE\sin\theta$ (cross)
$\tau = qr \times E = qrE\sin\theta$



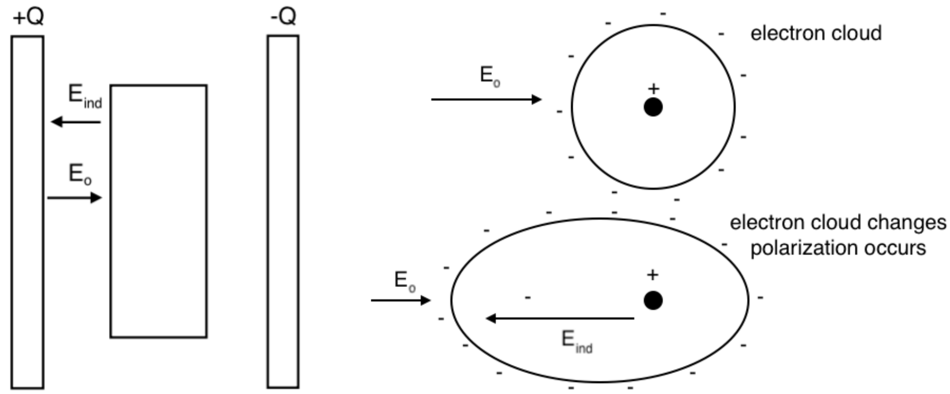
Unstable Equilibrium
When r || E, there is no torque
When r ⊥ E, there is max torque

$W = \tau\theta$
$dW = \tau d\theta$
$W = \int_{\theta_o}^{\theta_f} \tau d\theta = \int_{\theta_o}^{\theta_f} pE\sin\theta d\theta$
$W = -pE\cos\theta_f + pE\cos\theta_o$
If $\theta_o = 90^\circ$, then
$W = -pE\cos\theta_f = -pE\cos\theta$
$W = -p \bullet E$ (dot)



Stable Equilibrium

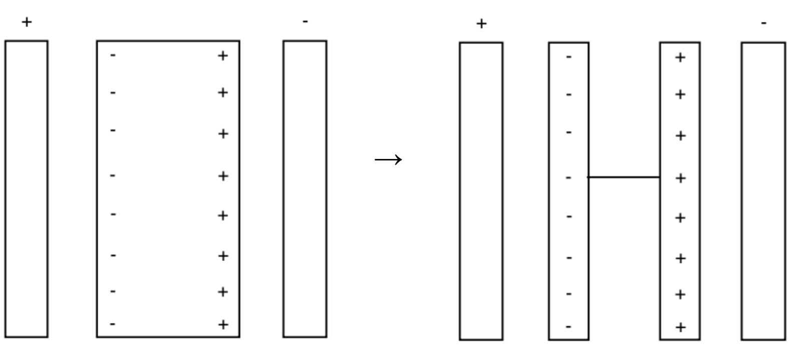
Capacitors & Dielectric



- Charged places create an E-field (E_o)
- Atoms on the insulator experience the E-field (E_o) and since the electrons aren't free to move (insulator), the electron cloud deforms and the atom is polarized
- Polarization creates an E-field (E_{ind} or $E_{induced}$) that counters E_o
- Total E-field in the gap is reduced: $E_{gap} = E_o - E_{ind}$
- Since $\Delta V = E_{gap}d$, a smaller E_{gap} means a smaller ΔV (potential difference)
- Since $C = \frac{Q}{\Delta V}$, a smaller ΔV means greater C (capacitance)
- Charge remains constant

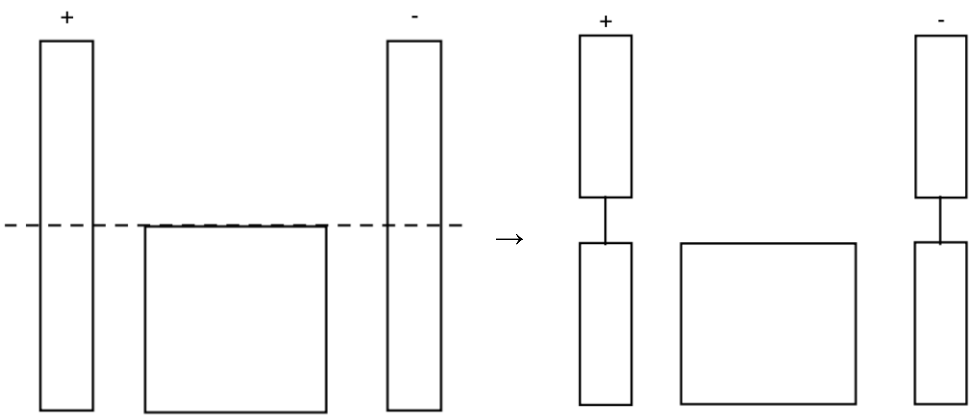
E_{gap} reduces by factor of 1/k
ΔV reduces by factor of 1/k
C increases by k

Conducting Dielectric within a Capacitor



- Electrons are able to flow freely
- Dielectric polarizes creating two individual capacitors connected in series
- Capacitance decreases per capacitor (defeats the purpose of a dielectric)
- $C' = \frac{C}{2}$

Partially Filled Capacitor



- We're able to cut capacitor in half
- This creates two individual capacitors connected in parallel
- C' = $\frac{C}{2}(1 + k)$

TIPS ON HOW TO SOLVE PROBLEMS:

1. IMPORTANT EQUATIONS/CONSTANTS
    a. $C = \frac{Q}{\Delta V}$
    b. $W = \frac{1}{2}\frac{Q^2}{C}$
    c. $W = \frac{1}{2}Q\Delta V$
    d. $W = \frac{1}{2}C\Delta V^2$
    e. τ = p x E = pEsinθ (for electric dipole)
    f. W = -p • E (for electric dipole)
    g. $\epsilon_k = \epsilon_o k$
2. Series Connections
    a. $Q_{total} = Q_1 = Q_2 = Q_3$
    b. $\Delta V_{total} = \Delta V_1 + \Delta V_2 + \Delta V_3$
    c. $C_{eq} = \frac{1}{\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}}$
3. Parallel Connections
    a. $Q_{total} = Q_1 + Q_2 + Q_3$
    b. $\Delta V_{total} = \Delta V_1 = \Delta V_2 = \Delta V_3$
    c. $C_{eq} = C_1 + C_2 + C_3$
4. Steps to solve Capacitance problems
    a. Find E_{gap} using Gauss' Law
    b. Find ΔV using $\Delta V = -\int_b^a E_{gap} \cdot ds$ and change E_{gap} domain if necessary
    c. Find Capacitance using $\frac{C}{\Delta V}$

5. Steps in reconnecting capacitors from series to parallel:
    a. Find C_{eq} (series)
    b. Find Q_{total} and Q on each capacitor
    c. Reconnect capacitors in parallel
    d. Find ΔV across each capacitor
    e. Find charge on each capacitor
6. Steps in reconnecting capacitors from parallel to series:
    a. Find C_{eq} (parallel)
    b. Find Q_1 and Q_2 to find Q_{total}
    c. Reconnect capacitors in series
    d. Find charge on each capacitor
    e. Find ΔV across each capacitor

UNIT 5: Current, Resistance, and Power

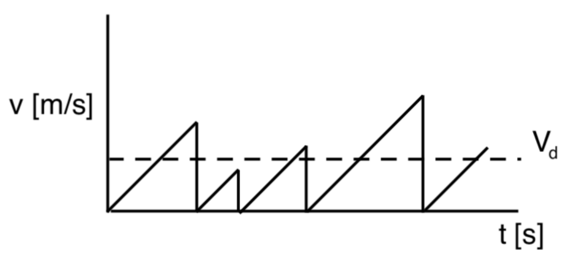
Fundamentals of Unit 5 Physics

Battery : device that holds two conducting terminals at a constant potential difference through chemical reaction

Current (I) [A or Amperes // C/S or Coulombs/Second] : flow of positive charge in a conductor
- $I = \frac{q}{\Delta t} = \frac{dq}{dt}$
- $I = \frac{nqA\Delta x}{t} = nqAV_d$
- $I = \int J \cdot dA$

Resistance (R) [Ω or Ohms] : the restriction of current through a device
- R = V/I

Drift Velocity of Charge Carriers (V_d) [m/s] : the average velocity of a free charge carrier (electrons) within a lattice structure under the Drude model of simulating charge carriers within a wire
- Assumption: Drift velocity is constant and the charge carriers are electrons
- Problem: There's an E-field in the wire causing the electrons to accelerate, why would velocity be constant?
- In reality: The electrons hit the atoms of the wire's lattice structure which we assume to cause the electron to lose all speed (violates conservation of momentum but this is only a model), then the electron speeds up again until it hits another atom
- Solution: The drift velocity is the average velocity of the electrons



Carrier Charge Density (n) [Charge Carriers/Volume] : density of charge carriers, usually electrons, within a given volume, usually a section of a wire of length L or Δx
- V = AΔx
    - V = volume
    - A = cross-sectional area
    - Δx = length

Total charge in a conducting wire (Q) [Coulombs] : charge within a certain volume or section of a wire
- $Q = nqA\Delta x$

Current Density (J) [Current/Area] : vector measurement of current per unit of area

- $J = \frac{I}{A} = nqV_d$

Conductivity of a Conductor (σ) [1 / Ohm Meter] : an intrinsic property that measure of a conductor's ability to conduct electricity

- Conductance is an extrinsic property measured in mhos

Resistivity of a Conductor (1/σ or ϱ) [Ohm Meter] : an intrinsic property that measure of a conductor's ability to resist electricity

- Resistance is an extrinsic property measured in ohms

Power (W) [Watts or J/C or Joules/Second] : rate of energy per second going through a conductor

- $P = I\Delta V = I^2R = \frac{\Delta V^2}{R}$

Circuit characteristics :

- Charges don't build up within a conductor
- No opposing E-field is created
- An E-field is created inside the conductor that causes the charge to flow (current)

Ohm's Law

1. In conductors: "current density and an electric field is established in a conductor when a potential difference is maintained across that conductor"
    a. $J = \sigma E$
2. In circuits: "Current in a circuit is directly proportional to the potential difference and inversely proportional to the resistance"
    a. $V = IR$

TIPS ON HOW TO SOLVE PROBLEMS:

1. IMPORTANT EQUATIONS/CONSTANTS
    a. $I = \frac{q}{\Delta t} = JA = \int J \cdot dA$
    b. Ohm's Law: $J = \sigma E$ and $V = IR$
    c. $\Delta V = EL \text{ or } Ed \ (distance)$
    d. $R = \frac{L}{\sigma A}$ (resistance in a wire/conductor)
    e. $P = I\Delta V = I^2R = \frac{\Delta V^2}{R}$
    f. $E = Pt = I\Delta Vt = I^2Rt = \frac{\Delta V^2}{R}t$

UNIT 6: Direct Connect (DC) Circuits and Resistor-Capacitor (RC) Circuits

Fundamentals of Unit 6 Physics

Circuit characteristics :

- Current is the positive flow of charge
- When we follow current through a resistor, we lose potential
- When we move from negative to positive terminals of a battery, we gain potential
- In ideal DC circuits, wires have no resistance
- Moving with current:
    - Through a resistor/capacitor, potential is lost
- Moving against current:
    - Through a resistor/capacitor, potential is gained
- When moving through a battery, potential is gain or lost based on the “longer line” or the final polarity because it is calculated using final - initial
    - (+) - (-) = $+\Delta V$ / (-) - (+) = $-\Delta V$

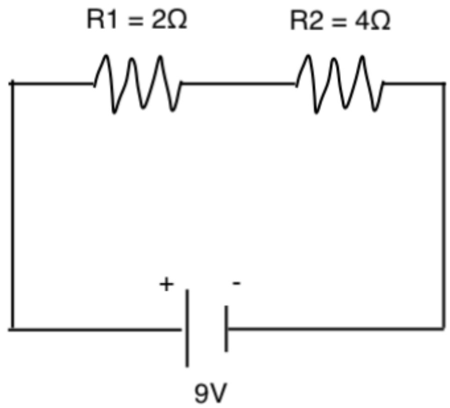
Electromotive Force (EMF) : electrostatic potential created due to chemical reaction in a battery or due to mechanical actions in a generator that explains the movement of electrons in a wire

- Not actually a force, but rather the potential difference that causes electrons to move
- Ideal Batteries: emf = $\Delta V_{battery}$
- Real Batteries: emf = $\Delta V_{battery} + Ir$
    - r = internal resistance of the battery
    - Ir = potential loss due to internal resistance of battery
        - This is what causes devices to heat up after a while

Time Constant (τ) [seconds or ΩF] : the length of time for a capacitor to fill with charge

- Small τ means the capacitor charges very quickly (defibrillator)
- Big τ means the capacitor charges very slowly

Series DC Circuits



Remember:

- Resistance is sum of all resistors
- Voltage is different
- Current is same

$R_{eq} = R_1 + R_2$

$I_{tot} = \Delta V/R_{eq}$

$\Delta V_1 = I_{tot}R_1$

$\Delta V_2 = I_{tot}R_2$

$\Delta V_{bat} = \Delta V_1 + \Delta V_2$

$R_{eq} = 6\Omega$

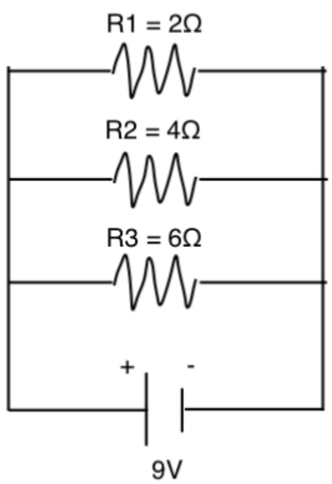
$I_{tot} = 1.5A$

$\Delta V_1 = 3V$

$\Delta V_2 = 6V$

$\Delta V_{bat} = 9V$ (KVL checks out)

Parallel DC Circuits



Remember:

- Resistance is $1/(1/R_1 + 1/R_2)$
- Voltage is same
- Current is different

$R_{eq} = \frac{1}{\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}}$

$\Delta V_{bat} = \Delta V_1 = \Delta V_2 = \Delta V_3$

$I_1 = \Delta V_{bat}/R_1$

$I_2 = \Delta V_{bat}/R_2$

$I_3 = \Delta V_{bat}/R_3$

$I_{tot} = I_1 + I_2 + I_3 = V_{bat}/R_{eq}$

$R_{eq} = 1.091\Omega$

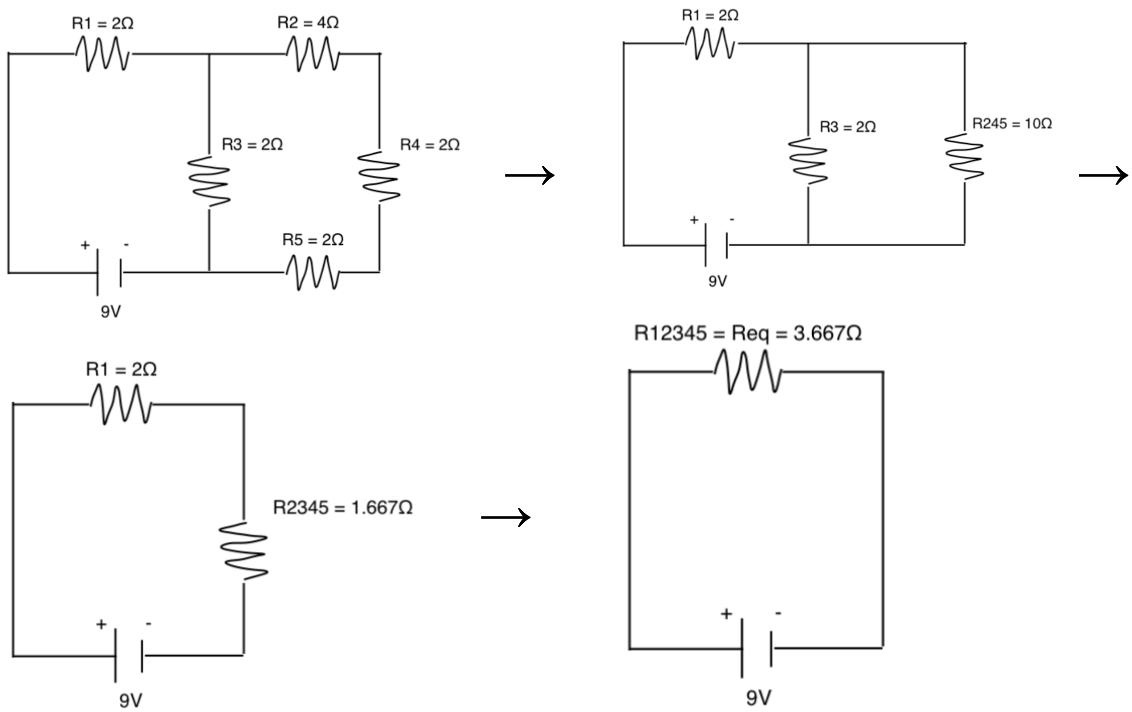
$I_1 = 4.5A$

$I_2 = 2.25A$

$I_3 = 1.5A$

$I_{tot} = 8.25A$

<u>Combinational DC Circuits</u>

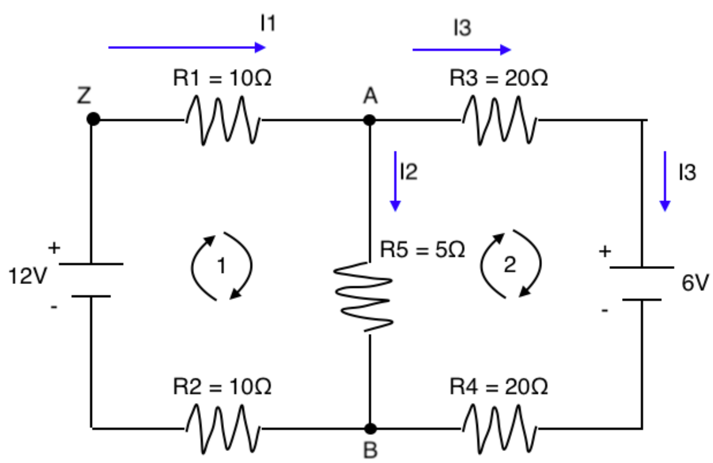


Steps:

1. Simplify the circuit by combining resistors in series then resistors in parallel then repeat until there's only one resistor and calculate R_{eq}
2. Work backwards from simple to complex circuit and find the voltage drop (ΔV) through each resistor
3. Find the current through each branch (not the same for all resistors as some are in parallel and some are in series)

Kirchhoff's Rules

1. Junction Rule / Kirchhoff's Current Law: current flowing into a junction is the same current flowing out of a junction
    a. Based on Conservation of Charge
2. Loop Rule / Kirchhoff's Voltage Law: when we move in a loop in a circuit, there is no change in potential difference ($\Delta V = 0$)
    a. Based on Conservation of Energy



JR: $I_1 = I_2 + I_3$

LR_1: $-\Delta V_{R1} - \Delta V_{R5} - \Delta V_{R2} + 12 = 0$
LR_1: $-I_1R_1 - I_2R_5 - I_1R_2 + 12 = 0$
LR_1: $-10I_1 - 5I_2 - 10I_1 + 12 = 0$
LR_1: $-20I_1 - 5I_2 + 12 = 0$

LR_2: $-\Delta V_{R3} - 6 - \Delta V_{R4} + \Delta V_{R5} = 0$
LR_2: $-I_3R_3 - 6 - I_3R_4 + I_2R_5 = 0$
LR_2: $-20I_3 - 6 - 20I_3 + 5I_2 = 0$
LR_2: $-40I_3 + 5I_2 - 6 = 0$

$I_1 = 0.52A$
$I_2 = 0.33A$
$I_3 = 0.19A$

Exponential Growth and Decay of Charge/Current/Voltage in RC Circuits

Growth:

- Capacitor is initially uncharged, but when the switch is connected, current begins flowing through the resistor and the capacitor begins charging
- Initially, the current doesn't account for the presence of the capacitor because it's uncharged so it acts as a highly conductive wire with no potential drop across it
    - Essentially, capacitor part of the circuit becomes a straight wire
- When the capacitor is fully charged, no current flows through the capacitor so after a long period, the capacitor acts as an open switch with maximum/E voltage across it
    - Essentially, capacitor part of the circuit is completely removed

$Q_{max} = EC$, $\tau = RC$

$Q(t) = EC(1 - e^{-t/RC})$ (increasing exponentially)

$Q(t) = Q_{max}(1 - e^{-t/RC})$ (increasing exponentially)

$Q(\tau) = Q_{max}(1 - e^{-\tau/RC}) = 0.63Q_{max}$

- τ is the amount of time it takes for charge to build up to 63.2% of total charge on the capacitor

$I_{max} = E/R$

$I(t) = \frac{E}{R} e^{-t/RC}$ (decreasing exponentially)

$I(t) = I_{max} e^{-t/RC}$ (decreasing exponentially)

$I(\tau) = I_{max}(e^{-\tau/RC}) = 0.368 I_{max}$

- τ is the amount of time it takes for current to decay down to 36.8% of total current on the capacitor

Voltage on Resistor: $\Delta V_R = I(t)R = E\, e^{-t/RC}$ (decreasing exponentially)

Voltage on Capacitor: $\Delta V_C = \frac{Q(t)}{C} = E(1 - e^{-t/RC})$ (increasing exponentially)

Decay:

- When a fully charged capacitor is connected to a resistor, the charges will move from positive to negative plate until capacitor is fully discharged
- Initially, switch is closed and current is maximum but decays exponentially when capacitor is discharged

$Q(t) = Q_{max} e^{-t/RC}$ (decreasing exponentially)

$I(t) = \frac{-Q_{max}}{RC} e^{-t/RC} = I_{max} e^{-t/RC}$ (decreasing exponentially)

Resistor	Capacitor
$\Delta V_R = I(t)R$ $\Delta V_R = \frac{Q_{max}}{C} e^{-t/RC}$ $\Delta V_{max} = \frac{Q_{max}}{C}$ $\Delta V_R(t) = \Delta V_{max} e^{-t/RC}$ (decreasing exponentially)	$\Delta V_C = \frac{Q(t)}{C}$ $\Delta V_C = \frac{Q_{max}}{C} e^{-t/RC}$ $\Delta V_{max} = \frac{Q_{max}}{C}$ $\Delta V_C(t) = \Delta V_{max} e^{-t/RC}$ (decreasing exponentially)

Notice how $\Delta V_R = \Delta V_C$ when circuit is disconnected from a battery and decaying

Energy stored in capacitors in RC Circuits

$EI - I^2 R + \frac{Q}{C} I = 0$

$I = \frac{dQ}{dt}$

$EI - I^2 R + \frac{Q}{C}\frac{dQ}{dt} = 0$

EI = rate of energy supplied by the battery

I^2R = rate of energy dissipated at the resistor

$\frac{Q}{C}\frac{dQ}{dt}$ = rate of energy stored in the capacitor u = energy stored in capacitor = $\frac{Q^2}{2C} = \frac{\epsilon_o E^2}{2} V$

- v = volume where E-field is present

Energy per unit of volume = $\frac{\epsilon_o E^2}{2}$

TIPS ON HOW TO SOLVE PROBLEMS:

1. IMPORTANT EQUATIONS/CONSTANTS
   a. GROWTH
      i. $Q_{max} = EC$, $\tau = RC$
      ii. $Q(t) = EC(1 - e^{-t/RC})$ (increasing exponentially)
      iii. $Q(t) = Q_{max}(1 - e^{-t/RC})$ (increasing exponentially)
      iv. $I_{max} = E/R$
      v. $I(t) = \frac{E}{R} e^{-t/RC}$ (decreasing exponentially)
      vi. $I(t) = I_{max} e^{-t/RC}$ (decreasing exponentially)
      vii. $\Delta V_R = I(t)R = E\, e^{-t/RC}$ (decreasing exponentially)
      viii. $\Delta V_C = \frac{Q(t)}{C} = E(1 - e^{-t/RC})$ (increasing exponentially)
   b. DECAY
      i. $Q(t) = Q_{max} e^{-t/RC}$ (decreasing exponentially)
      ii. $I(t) = \frac{-Q_{max}}{RC} e^{-t/RC} = I_{max} e^{-t/RC}$ (decreasing exponentially)
   c. u = energy stored in capacitor = $\frac{Q^2}{2C} = \frac{\epsilon_o E^2}{2} V$
2. Moving with current:
   a. Through a resistor/capacitor, potential is lost
3. Moving against current:
   a. Through a resistor/capacitor, potential is gained
4. When moving through a battery, potential is gain or lost based on the "longer line" or the final polarity because it is calculated using final - initial
   a. (+) - (-) = +ΔV / (-) - (+) = -ΔV
5. Capacitor is a wire when switch is closed, then becomes completely gone after a long time as it's full of charge with no current through it
6. Series Circuits:
   a. Resistance is sum of all resistors
   b. Voltage is different
   c. Current is same
7. Parallel Circuits:
   a. Resistance is $1/(1/R_1 + 1/R_2)$
   b. Voltage is same
   c. Current is different

UNIT 7: Magnetic Fields

Fundamentals of Unit 7 Physics

Magnetic Field (B-Field) [T or Tesla]: a field that is created by the movement of charges relative to each other

- A magnetic field is generated when there's a movement of charge (current)
  - Direction of B-field is how the fingers curl when performing RHR and thumb is in direction of current
- 1 Tesla = 10,000 Gauss
- Always flows from North to South pole
- There is <u>NO</u> work done by a <u>STATIC</u> B-field on a <u>FREE</u> moving charge
  - A B-field just causes the charge to move in a circle at constant velocity

Magnetic Force (F_B) [N] : force on a charged particle due to a magnetic field

- $F_B = q(v \times B)$ (formula used for free moving charges in a uniform magnetic field)
  - q = amount of charge on the particle
  - v = velocity of the charge
  - B = magnetic field
- $F_B = I(L \times B)$ (formula used for current carrying wires in a uniform magnetic field)
  - I = current
  - L = length vector (magnitude is the wire length and direction is the same as current))
  - B = magnetic field
- $|F_B| = qvB\sin\theta$
  - θ is angle between v and B
- Direction of F_B is given by Right-Hand-Rule (RHR)
  - Index finger: B-field
  - Thumb: velocity of charge or current
  - Middle finger: F_B
- ⊙ = out of the page (positive)
- ⓧ = into the page (negative)

Lorentz Force (F_L or $F_B + F_E$) [N] : total force on a charged particle, including the electrostatic force by an E-field and the magnetic force by a magnetic field

- $F_L = F_E + F_B = qE + q(v \times B)$

Magnetic Dipole Moment (μ_B) : the maximum amount of torque caused by magnetic force on a dipole that arises per unit value of surrounding magnetic field in vacuum

- $\mu_B = IA$
  - I = current
  - A = area vector

Electron-Volt (eV) : unit of energy for small scales of charge (like electrons), the amount of potential energy gained when an electron moves through a potential difference of 1V

- $\Delta U = q\Delta V = (1.6 \times 10^{-19})(1) = 1.6 \times 10^{-19}$ J or 1 eV

Cyclotron : a charged particle accelerator

- A charged particle is placed inside a magnetic field causing the particle to move in a circle
- The particle traverses a gap with a potential difference due to a voltage source which accelerates the particle
- The voltage source is powered by an alternating current so the potential difference will always cause the particle to "fall" and accelerate even further

Cyclotron Frequency (ω) [Hertz or Hz] : special case of angular frequency regarding a charged particle moving through a magnetic field

- $\omega = \frac{qB}{m}$
    - q = charge on particle
    - B = magnetic field
    - m = mass of particle

Permeability of free space (μ_o) [Tesla meter / Ampere] : constant used to quantify the strength of a magnetic field emitted by an electric current

- $\mu_o = 4\pi \times 10^{-7}$

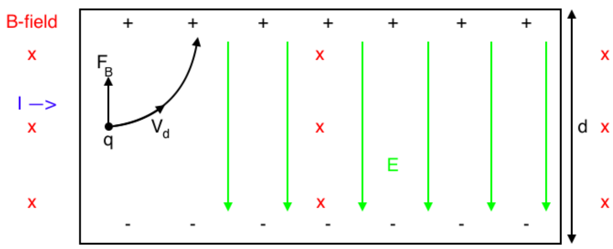
Magnetic Flux (Φ_B) [T m^2 or Webber (Wb)] :

- $\Phi_B = \oint B \bullet dA$
    - B = magnetic field
    - dA = differential area element

Solenoid : coil of wire that is lightly wound and carrying a current, creating a uniform magnetic field within the inside of the solenoid

Toroid : similarly to a solenoid, a coil of wire is lightly wound about a circular pattern and carrying a current, creating a uniform magnetic field within the inside of the toroid

Hall's Effect



- Current flows through the conducting strip
- A B-field is going in and through the strip
- Positive charges accumulate to the top of the magnetic strip due to F_B
- Polarization creates a potential difference (ΔV_{Hall}) and E-field from the top to bottom of the strip
- Charges feel a magnetic force and electrostatic force

Charges that are allowed to flow through the conducting strip have achieved equilibrium (forces cancel)

$F_E = F_B$

$qE = qV_dB$

$E = V_dB$

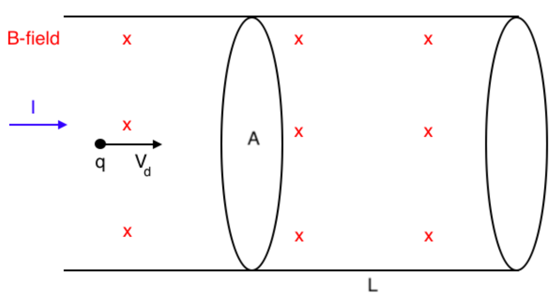
Since $\Delta V = Ed$: Since $V_d = \frac{I}{nqA}$:

$\Delta V_{Hall} = V_dBd$ $\Delta V_{Hall} = \frac{I}{nqA}Bd$

$\Delta V_{Hall} = \frac{I}{nqA}Bd = V_dBd$

Notice: Charges that can flow through the conducting strip require a speed of $V_d = \frac{E}{B}$ (this is called a velocity selector because charges of a certain speed are only allowed to flow through)

Force on a current carrying wire in a uniform B-field



This section is for proving how $F_B = I(L \times B)$

Remember:

- $Q = nqAL$
- $I = nqAV_d$

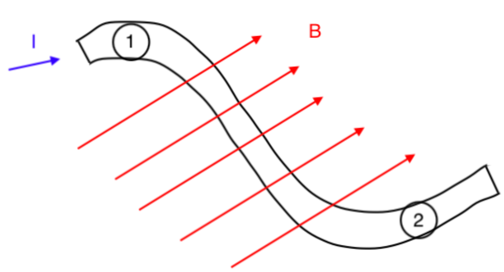
$F_B = Q(V_d \times B)$

$F_B = nqAL(V_d \times B)$

$F_B = I(L \times B)$

L is the wire length and its direction is same as current

Force on a curved current carrying wire in a uniform B-field



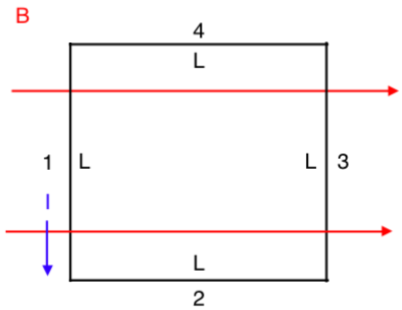
$F_B = I(L \times B)$

$dF_B = I(ds \times B)$

$F_B = I\oint_1^2 ds \times B$ (contour/line integral)

Note: contour integrals are part of vector calculus which won't be explored in AP Physics C

Force on a rectangular current carrying wire in a uniform B-field

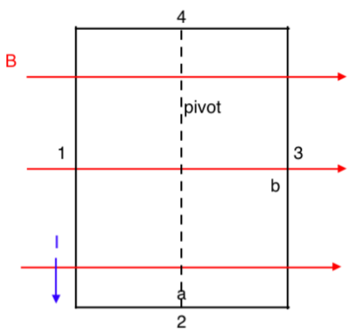


Approach: split each side length into its own section and superimpose

$F_B = I(L \times B)$

1. $F_{B1} = I(LB\sin 90^\circ)\ \check{k} = ILB\ \check{k}$
2. $F_{B2} = I(LB\sin 0^\circ)\ \check{k} = 0\ \check{k}$
3. $F_{B3} = I(LB\sin 90^\circ)\ -\check{k} = ILB\ -\check{k}$
4. $F_{B4} = I(LB\sin 0^\circ)\ \check{k} = 0\ \check{k}$

$\Sigma F = ILB - ILB = 0$



The addition of a pivot in the middle will cause the rectangular current carrying wire to rotate

1. $F_{B1} = I(bB\sin 90^\circ)\ \check{k} = IbB\ \check{k}$
2. $F_{B2} = I(aB\sin 0^\circ)\ \check{k} = 0\ \check{k}$
3. $F_{B3} = I(bB\sin 90^\circ)\ -\check{k} = IbB\ -\check{k}$
4. $F_{B4} = I(aB\sin 0^\circ)\ \check{k} = 0\ \check{k}$

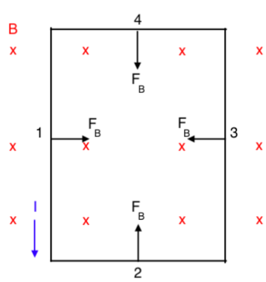
$\Sigma F = 0$

$\tau = r \times F_B$

1. $\tau_1 = rF\ \hat{j} = a/2\ IbB\ \hat{j}$
2. $\tau_3 = rF\ \hat{j} = a/2\ IbB\ \hat{j}$

$\Sigma\tau = r \times F_B$

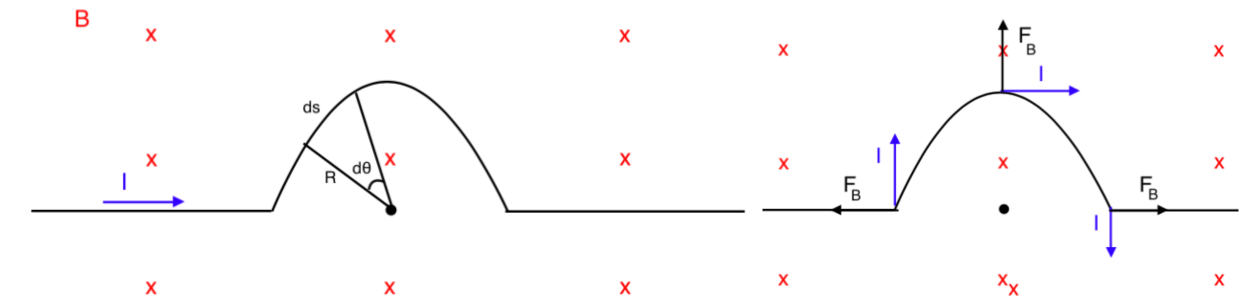
$\Sigma\tau = abIB\ \hat{j} = \mu_B \times B\ \hat{j}$ (μ_B = magnetic dipole moment, $\check{k}$)



A B-field going into ($-\check{k}$) and through the wire will cause the rectangular current carrying wire to contract

A B-field going out ($\check{k}$) and through the wire will cause the rectangular current carrying wire to expand

Force on a semi-circular current carrying wire in a uniform B-field



$F_B = I(L \times B)\,\hat{r}$ [general eq.]

$dF_B = I(ds \times B)\,\hat{r}$ [turn into differential]

$dF_B = I(Rd\theta * B)\,\hat{r}$ [substitute ds = rdθ]

$dF_B = IBR\cos\theta d\theta\,\hat{\imath} + IBR\sin\theta d\theta\,\hat{\jmath}$ [turn $\hat{r}$ into $\hat{\imath}$ and $\hat{\jmath}$ components]

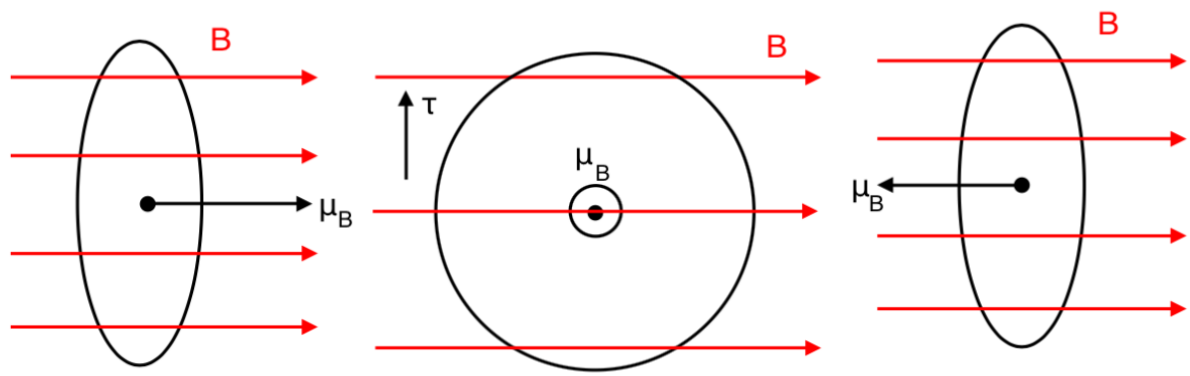
$F_B = IBR\int_0^\pi \cos\theta d\theta\,\hat{\imath} + IBR\int_0^\pi \sin\theta d\theta\,\hat{\jmath}$ [apply limits]

$F_B = IBR(\sin\theta)_0^\pi\,\hat{\imath} + IBR(-\cos\theta)_0^\pi\,\hat{\jmath}$ [integrate]

$F_B = IBR(0 - 0)\,\hat{\imath} + IBR(-1 - (-1))\,\hat{\jmath}$ [evaluate]

$F_B = 2IBR\,\hat{\jmath}$ [simplify]

Torque on a circular current loop & magnetic dipole moment



τ = 0
KE = 0
U = min (negative)
Stable Equilibrium

τ = max
KE = max
U = 0

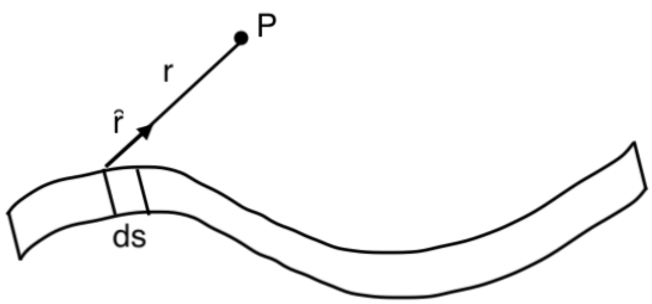
τ = 0
KE = 0
U = max
Unstable equilibrium

Potential Energy of a magnetic dipole moment in a uniform B-field (drawing parallels)

Magnetic Dipole	**Electrostatic Dipole**
$\mu_B = IA$ $\tau = \mu_B \times B$ $U_B = -\mu_B * B$ $U_B = -IA * B$	$p = qd$ $\tau = p \times E$ $U_E = -p * E$ $U_E = -qd * E$

Biot-Savart's Law

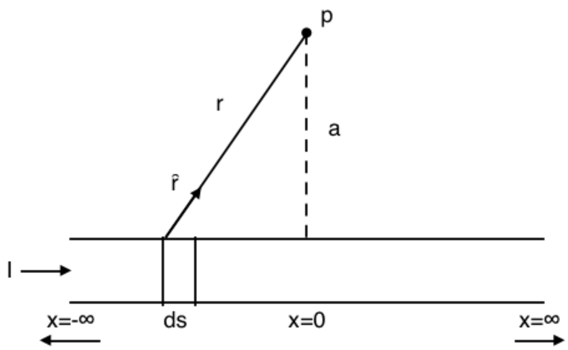
Law that tells us the magnetic field due to a current some distance away from said current
- Similar to Coulomb's Law with distributed charges

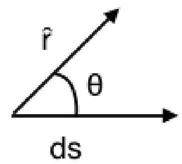


$$dB_p = \frac{\mu_o I}{4\pi} * \frac{ds \times \hat{r}}{|r|^2}$$

- ds = differential wire element
- r = distance from ds to point P or point of interest
- $\hat{r}$ = unit vector along r towards point P or point of interest
- μ_o = permeability of free space = $4\pi \times 10^{-7}$ T m / A

1. **<u>Biot-Savart's Law on an infinitely long wire</u>**





$r = \sqrt{a^2 + x^2}$

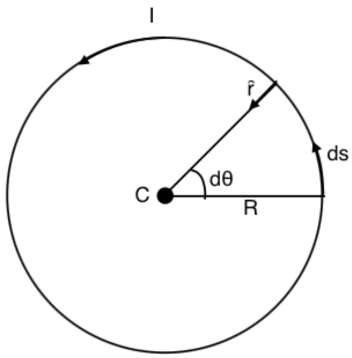
$\sin\theta = \frac{a}{r} = \frac{a}{\sqrt{a^2+x^2}}$

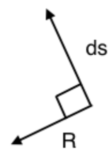
ds = dx

ds x r̂ = ds * 1 * sinθ ǩ = sinθds ǩ

$dB_p = \frac{\mu_o I}{4\pi} * \frac{ds \times \hat{r}}{\lvert r\rvert^2}$	[general eq.]
$dB_p = \frac{\mu_o I}{4\pi} * \frac{\sin\theta ds}{\lvert r\rvert^2}\ \check{k}$	[sub ds x r̂ = sinθds ǩ]
$dB_p = \frac{\mu_o I}{4\pi} * \frac{\frac{a}{\sqrt{a^2+x^2}}dx}{\lvert r\rvert^2}\ \check{k}$	[sub $\sin\theta = \frac{a}{\sqrt{a^2+x^2}}$ and ds = dx]
$dB_p = \frac{\mu_o I}{4\pi} * \frac{\frac{a}{\sqrt{a^2+x^2}}dx}{a^2+x^2}\ \check{k}$	[sub $r = \sqrt{a^2 + x^2}$]
$dB_p = \frac{\mu_o Ia}{4\pi} * \frac{dx}{(a^2+x^2)^{3/2}}\ \check{k}$	[take out constant and combine terms]
$B_p = \frac{\mu_o Ia}{4\pi} * \int_{-\infty}^{\infty} \frac{dx}{(a^2+x^2)^{3/2}}\ \check{k}$	[apply limits from -∞ to ∞]
$B_p = \frac{\mu_o Ia}{4\pi} * \left(\frac{x}{a^2(a^2+x^2)^{1/2}}\right)_{-\infty}^{\infty}\ \check{k}$	[integrate (integral solution given)]
$B_p = \frac{\mu_o Ia}{4\pi} * \left(\frac{\infty}{a^2\infty} + \frac{\infty}{a^2\infty}\right)\ \check{k}$	[evaluate from -∞ to ∞, $a^2 + x^2 = x^2$]
$B_p = \frac{\mu_o Ia}{4\pi} * \left(\frac{2}{a^2}\right)\ \check{k}$	[simplify]
$B_p = \frac{\mu_o I}{2\pi a}\ \check{k}$	[simplify]

2. Biot-Savart's Law on a circular current loop at loop's center





ds x r̂ = ds * 1 * sin90˚ = ds $\check{k}$ (r̂ always ⊥ ds)
ds = Rdθ

$$dB_p = \frac{\mu_o I}{4\pi} * \frac{ds \, x \, \hat{r}}{|r|^2}$$ [general eq.]

$$dB_p = \frac{\mu_o I}{4\pi} * \frac{ds}{|r|^2} \check{k}$$ [sub ds x r̂ = ds $\check{k}$]

$$dB_p = \frac{\mu_o I}{4\pi R^2} * ds \, \check{k}$$ [sub $r^2 = R^2$ and take out R constant]

$$dB_p = \frac{\mu_o I}{4\pi R^2} * R d\theta \, \check{k}$$ [sub ds = Rdθ]

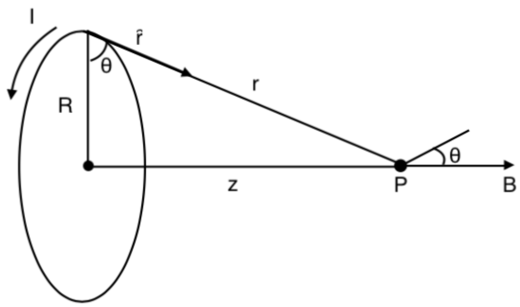
$$dB_p = \frac{\mu_o I}{4\pi R} * d\theta \, \check{k}$$ [cancel R and simplify]

$$B_p = \frac{\mu_o I}{4\pi R} * \int_0^{2\pi} d\theta \, \check{k}$$ [apply limits from 0 to 2π]

$$B_p = \frac{\mu_o I}{4\pi R} * 2\pi \, \check{k}$$ [integrate and evaluate]

$$B_p = \frac{\mu_o I}{2R} \check{k}$$ [simplify]

3. **Biot-Savart's Law on a circular current loop at a distance from loop's center along z-axis**



ds x r̂ = ds * 1 * sin90° = ds ǩ (r̂ always ⊥ ds)
Note: Since x/y-components are canceled out due to symmetry, only z-components will be superimposed:
ds ǩ → ds cosθ ǩ

$r = \sqrt{R^2 + z^2}$

$\cos\theta = \frac{R}{r} = \frac{R}{\sqrt{R^2+z^2}}$

$dB_p = \frac{\mu_o I}{4\pi} * \frac{ds \times \hat{r}}{|r|^2}$ [general eq.]

$dB_p = \frac{\mu_o I}{4\pi} * \frac{ds}{|r|^2} \cos\theta\ \check{k}$ [sub ds x r̂ = ds cosθ ǩ]

$dB_p = \frac{\mu_o I}{4\pi} * \frac{ds}{R^2+z^2} * \frac{R}{\sqrt{R^2+z^2}}\ \check{k}$ [sub $\cos\theta = \frac{R}{\sqrt{R^2+z^2}}$ and $r = \sqrt{R^2 + z^2}$]

$B_p = \frac{\mu_o IR}{4\pi(R^2+z^2)^{3/2}} * \oint ds\ \check{k}$ [combine like terms and integrate ds]

$B_p = \frac{\mu_o IR}{4\pi(R^2+z^2)^{3/2}} * 2\pi R\ \check{k}$ [$\oint ds = 2\pi R$ (path of ds)]

$B_p = \frac{\mu_o IR^2}{2(R^2+z^2)^{3/2}}\ \check{k}$ [simplify]

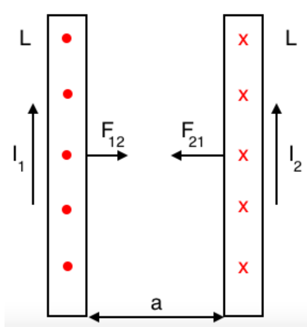
If z = 0, then $R^2 + z^2 = R^2$

$B_p = \frac{\mu_o I}{2R}\ \check{k}$ (matches example 2)

If z = ∞, then $R^2 + z^2 = z^2$

$B_p = \frac{\mu_o IR^2}{2z^3}\ \check{k}$ (B varies by ⅓ as z increases)

Force between two current carrying wires



B_{xy}: B-field created by wire x @ location of wire y
B_{12}: B-field created by wire 1 @ location of wire 2
B_{21}: B-field created by wire 2 @ location of wire 1

$B_{12} = \frac{\mu_o I_1}{2\pi a}\ -\check{k}$

$F_{12} = I_1(L \times B_{21})\ \hat{i}$

$F_{12} = I_1 L \frac{\mu_o I_2}{2\pi a}\ \hat{i}$

$B_{21} = \frac{\mu_o I_1}{2\pi a}\ \check{k}$

$F_{21} = I_2(L \times B_{12})\ -\hat{i}$

$F_{21} = I_2 L \frac{\mu_o I_1}{2\pi a}\ -\hat{i}$

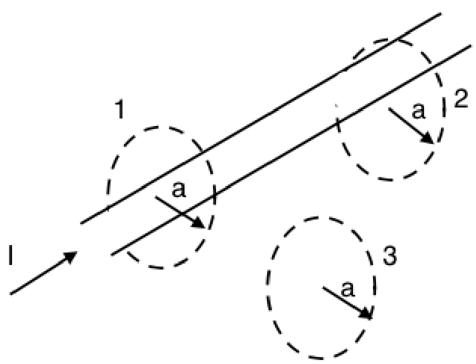
The two wires will come together when the current is parallel
However, if the current was antiparallel, the wires would repel each other

Ampere's Law

The magnetic field along a closed path equals the product of μ_o (permeability of free space) and the total current through the area enclosed by the path

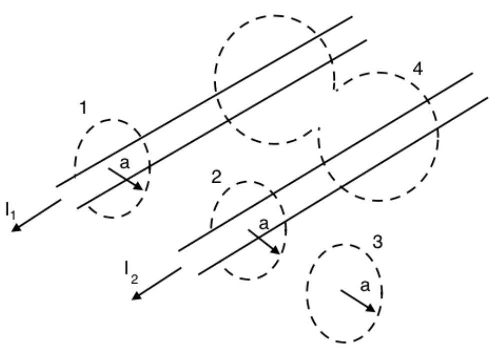
- $\oint B \bullet ds = \mu_o I_{total}$
- Similar to Gauss' Law, used for highly symmetric geometries

1. **Ampere's Law on an infinitely long wire**



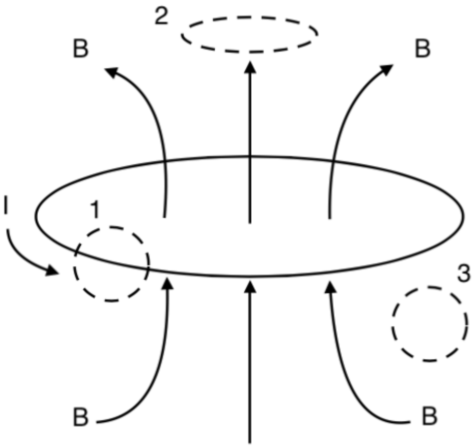
$\oint B_1 \bullet ds = \mu_o I_{total}$	[general eq.]
$\oint B_1 ds = \mu_o I_{total}$	[B ∥ ds, remove dot]
$B_1 \oint ds = \mu_o I_{total}$	[B constant]
$B_1(2\pi a) = \mu_o I_{total}$	[integrate ds]
$B_1 = \frac{\mu_o I_{total}}{2\pi a}$	[simplify]
$\oint B_2 \bullet ds = 0$	[no current in loop]
$\oint B_3 \bullet ds = \mu_o I_{total}$	[vector calculus needed]

2. **Ampere's Law on two wires**



$\oint B_1 \bullet ds = \mu_o I_1$	[only I_1 in loop]
$\oint B_2 \bullet ds = \mu_o I_2$	[only I_2 in loop]
$\oint B_3 \bullet ds = 0$	[no I in loop]
$\oint B_4 \bullet ds = \mu_o [I_1 + I_2]$	[I_1 and I_2 in loop]

3. **Ampere's Law on an a looped wire**



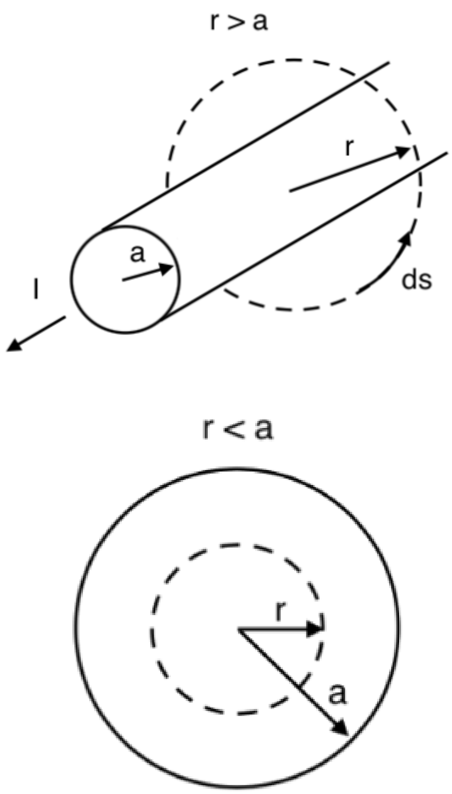
$\oint B_1 \bullet ds = \mu_o I_1$ [only I in loop]

Note: Can't actually perform integral because B isn't constant
Must use Biot-Savart's Law

$\oint B_2 \bullet ds = 0$ [no I in loop]

$\oint B_3 \bullet ds = 0$ [no I in loop]

4. **Ampere's Law on an a thick wire**



Outside the thick wire (r > a):

$\oint B \bullet ds = \mu_o I_{total}$ [general eq.]

$\oint B ds = \mu_o I$ [B || ds, remove dot]

$B \oint ds = \mu_o I$ [B constant]

$B(2\pi a) = \mu_o I$ [integrate ds]

$B = \frac{\mu_o I}{2\pi r}$ [simplify]

Inside the thick wire (r < a):
To get the amount of total current enclosed, we need a ratio:

$I_{total} = current\ density * area = \frac{I}{\pi a^2} * \pi r^2 = \frac{I r^2}{a^2}$

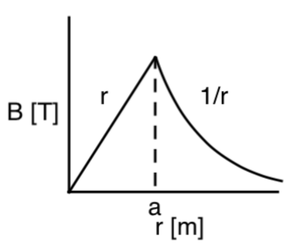
$\oint B \bullet ds = \mu_o I_{total}$ [general eq.]

$\oint B ds = \mu_o I_{total}$ [B || ds, remove dot]

$B \oint ds = \mu_o I_{total}$ [B constant]

$B(2\pi a) = \mu_o I_{total}$ [integrate ds]

$B(2\pi a) = \mu_o \frac{I r^2}{a^2}$ [replace $I_{total} = \frac{I r^2}{a^2}$]



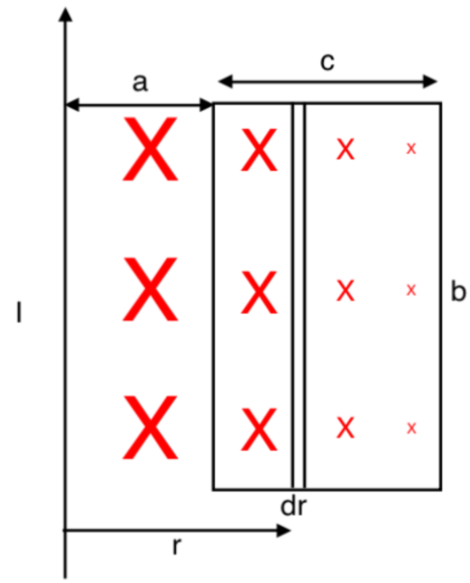
$B = \frac{\mu_o I r}{2\pi a^2}$ [simplify]

Gauss' Law for Magnetism

States that there's no magnetic monopoles

- $\oint B \bullet dA = 0$
- Magnetostatic flux through an enclosed surface is always 0

1. **Magnetic Flux through a rectangular loop**



$$\Phi_B = \oint B \bullet dA$$ [general eq.]

$$d\Phi_B = \frac{\mu_o I}{2\pi r} * bdr$$ [replace B and dA]

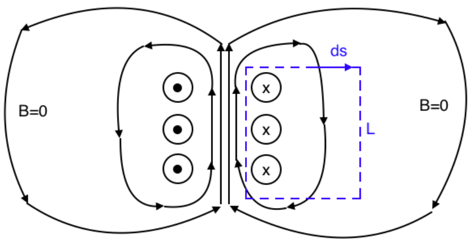
$$\Phi_B = \int_a^{a+c} \frac{\mu_o Ibdr}{2\pi r}$$ [apply limits]

$$\Phi_B = \frac{\mu_o Ib}{2\pi} \int_a^{a+c} \frac{dr}{r}$$ [take out constants]

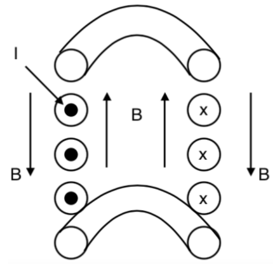
$$\Phi_B = \frac{\mu_o Ib}{2\pi} ln\left|\frac{a+c}{a}\right|$$ [integrate and evaluate]

<u>Creating a Uniform B-field</u>

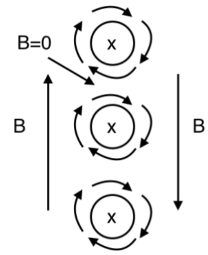
Solenoid Model: multiple current loops stacked on top of each other (this simplifies reality)



Large scale (entire solenoid)



Medium scale (inside solenoid)



Small scale (between each loop)

N = number of loops — I = current in each loop

n = loop density = N/L — L = length of ds or length of selected area of N number of loops

Understand: Only side length "L" inside solenoid will create B-field

$\oint B \bullet ds = \mu_o I_{total}$ [general eq.]

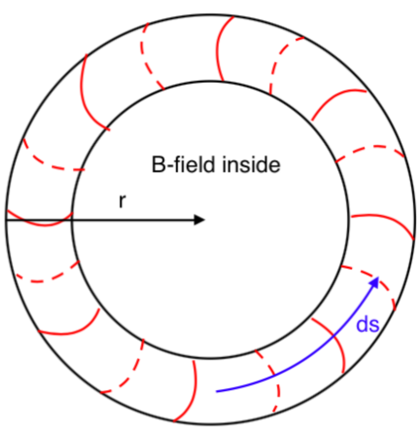
$B \oint ds = \mu_o IN$ [I_{total} = IN = current * # of loops]

$BL = \mu_o IN$ [$\oint ds$ = L from "Understand"]

$B = \mu_o I \frac{N}{L}$ [simplify]

$B = \mu_o In$ [n = N/L = loop density]

Toroid:



$\oint B \bullet ds = \mu_o I_{total}$ [general eq.]

$B \oint ds = \mu_o IN$ [I_{total} = IN]

$B(2\pi r) = \mu_o IN$ [$\oint ds$ = 2πr from circumference]

$B = \frac{\mu_o IN}{2\pi r}$ [simplify]

TIPS ON HOW TO SOLVE PROBLEMS:

1. IMPORTANT EQUATIONS/CONSTANTS
   a. $F_B = q(V_d \times B) = I(L \times B)$
   b. $\mu_o = 4\pi \times 10^{-7}$ (Permeability of free space)
   c. $\Delta V_{Hall} = \frac{I}{nqA}Bd = V_d Bd$ (recall $I = nqAV_d$)
   d. $V_d = \frac{E}{B}$ (velocity selector)
   e. $1\ eV = 1.6 \times 10^{-19}\ J$ (1 electron volt)
   f. $dB_p = \frac{\mu_o I}{4\pi} * \frac{ds \times \hat{r}}{|r|^2}$ (Biot-Savart's Law)
   g. $\oint B \bullet ds = \mu_o I_{total}$ (Ampere's Law)
   h. I_{total} = current density * area (only applies to within a current carrying wire)
      i. $I_{total} = \frac{I}{total\ area} * area\ enclosed$
   i. $\Phi_B = \oint B \bullet dA$ (Magnetic flux)
   j. $\mu_B = IA$ (magnetic dipole moment)
   k. $\tau = \mu_B \times B$ (torque on magnetic dipole moment)
   l. $\tau = r \times F_B$ (general torque caused by F_B)
   m. $\omega = \frac{qB}{m}$ (cyclotron frequency)
   n. $B_p = \frac{\mu_o I}{2\pi a}$ (Magnetic field "a" distance away from current/wire)
   o. $B_{solenoid} = \mu_o nI = \mu_o NI / L$ (B-field inside a solenoid)
2. RHR:
   a. Thumb points in direction of current, fingers curl in direction of B-field
      i. You can also curl finders in direction of current and thumb points in direction of B-field, useful for current carrying loops
   b. Index finger points in direction of B-field, Thumb is in direction of V_d or L, Middle finger is in direction of F_B
      i. F = A(B x C), B = thumb, C = index, F = middle
3. Ampere's Law should only be used in completely symmetrical situations (usually only in one plane) while Biot-Savart's Law should be used in all other situations
4. $\hat{r}$ is in direction pointing towards point P or point of interest
5. Recall formula for final velocity (V_f) for a particle moving through a potential difference
   a. $V_f = \sqrt{\frac{2q\Delta V}{m}}$
6. Solving problems where magnetic field causes a particle to rotate in a circle
   a. $F_B = mv_d^2 / R$
   b. $q(v_d \times B) = mv_d^2 / R$
   c. $qv_dB = mv_d^2 / R$
   d. $qB = mv_d / R$
   e. Some questions may ask for KE: $KE = \frac{1}{2}mv_d^2$

7. Wires parallel to each other with current going in the SAME direction will attract (F_B towards each other)
8. Wires parallel to each other with current going in OPPOSIˇE directions will repel (F_B away from each other)
9. If question gives positive charges moving in a certain direction, treat it as current to help you understand direction of Magnetic Force or B-field

UNIT 8: Electromagnetic Induction/Faraday's Law

Fundamentals of Unit 8 Physics

Electromagnetic Induction : in Unit 7, we converted electricity into magnetism and B-fields, now we're converting magnetism into electricity

Inductor : a device that diminishes current and fights the current by producing an induced current in the opposite direction

- Purpose is to smooth out a sudden jump in current within a circuit as it grows (this spike can cause damage to the circuit)
- Ex: Solenoid, wireless chargers

Inductance (L) [Henrys or H] : characteristic that causes an inductor to diminish current with back emf

- $L = -\frac{\varepsilon_L}{\frac{dI}{dt}} = N\frac{\Phi_B}{I}$

ε_L **or "Back EMF"** : the opposing emf generated in an inductor as a result of current growth or decay

- $\varepsilon_L = -L\frac{dI}{dt}$

Faraday's Law

A changing magnetic flux through a coil of wires induces an electromotive force "emf" according to:

$\varepsilon = -\frac{d\Phi_B}{dt} = -\frac{d}{dt}\oint B \bullet dA = \oint E \bullet ds$ $\Phi_B = \oint B \bullet dA$ [Webbers or Wb]

- Induction depends on geometry
- $\varepsilon(solenoid) = -N\frac{d\Phi_B}{dt}$ [N = number of loops]

Lenz's Law

Induced currents create magnetic fields that oppose the emf or the rate of change of magnetic flux, that created the induced current in the first place

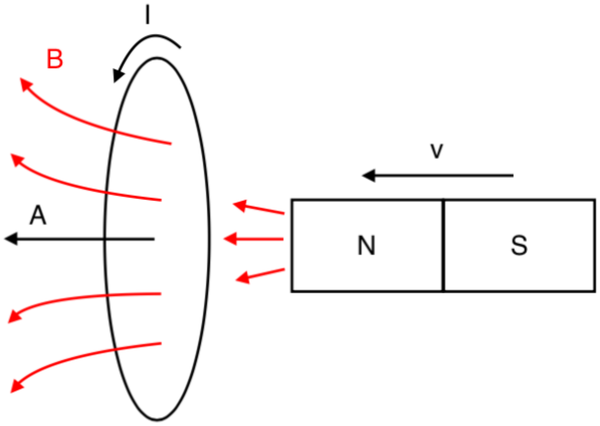
- A way to get the direction of induced current without thinking about the area vector
- Conserves energy in such a way where the direction of induced current doesn't create free energy
- Eddy Currents = currents within a solid and opposes motion and dissipates heat

Changing B-FIELD creates INDUCED STATIC E-FIELD

- Induced static E-fields don't have a start or stop point in space
- Generated due to changing B-field
- Not fixed in time
- Appear and disappear constantly
- Not conservative

Case Studies for Electromagnetic Induction/Faraday's Law

1. Loop with increasing flux (Magnet = N/S going left)



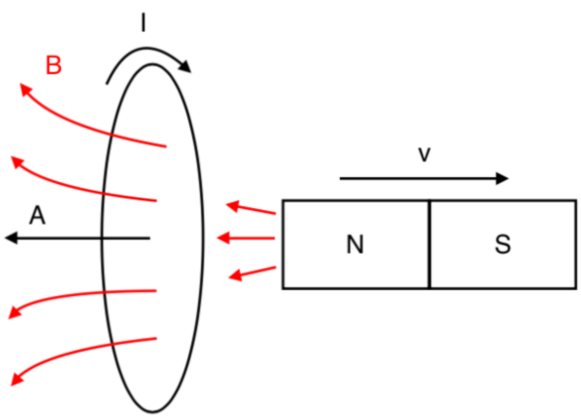
B is going left and increasing

Current generated so that $B_{induced}$ is going left

Note: Current is created in such a way that it creates an induced magnetic field opposite to the increase in magnetic flux, so induce magnetic field would be going to the right (which current would correlate to)

$\frac{d\Phi_B}{dt}$ (positive) so ε (negative)

2. Loop with decreasing flux (Magnet = N/S going right)

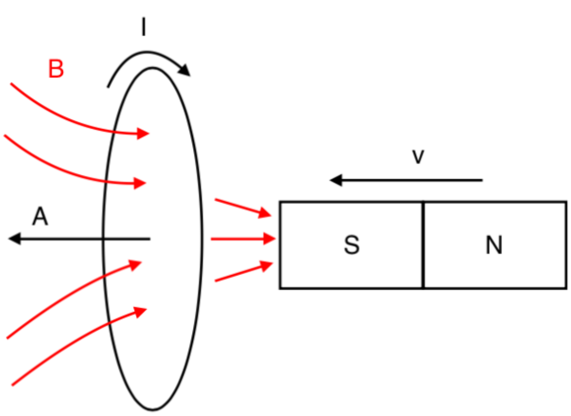


B is going left and decreasing

Current generated so that $B_{induced}$ is going left

$\frac{d\Phi_B}{dt}$ (negative) so ε (positive)

3. Loop with decreasing flux (Magnet = S/N going left)



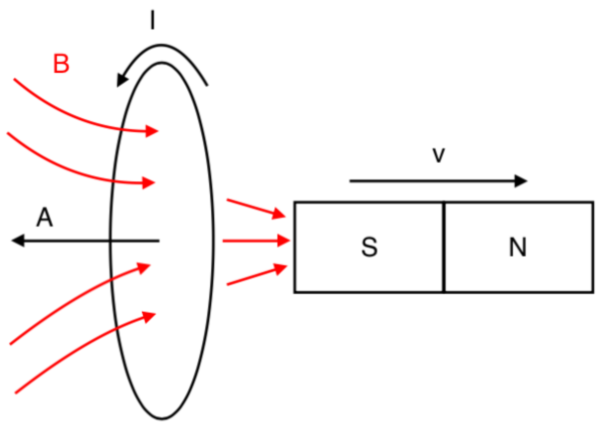
B is going right and increasing

Current generated so that $B_{induced}$ is going left

Note: Flux is increasing in the negative direction means that $\frac{d\Phi_B}{dt}$ is negative (area vector opposite to B-field or cos180˚ = -1)

$\frac{d\Phi_B}{dt}$ (negative) so ε (positive)

4. Loop with increasing flux (Magnet = S/N going right)

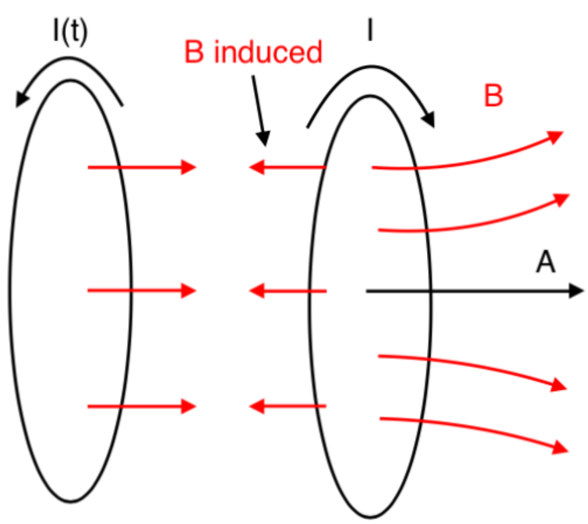


B is going right and decreasing

Current generated so that $B_{induced}$ is going right

$\frac{d\Phi_B}{dt}$ (positive) so ε (negative)

5. Time varying current loop near another loop

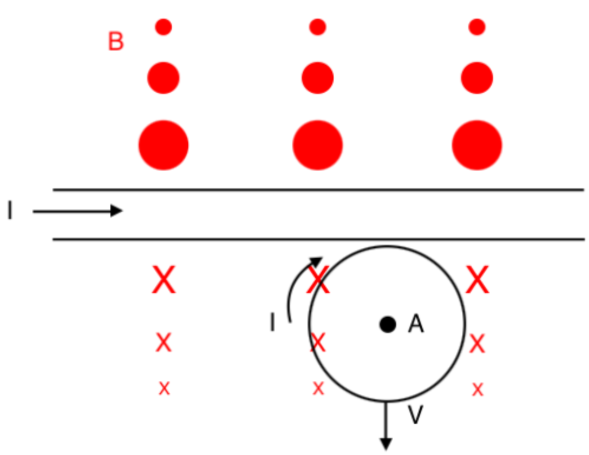


I(t) is increasing so B going right is increasing

Current (I) is generated to create $B_{induced}$ to oppose B

I(t) is increasing so $\frac{d\Phi_B}{dt}$ (positive) so ε (negative)

6. Moving loop near a wire

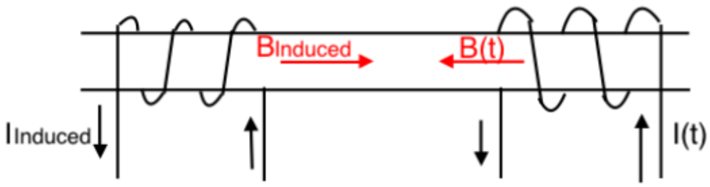


B going in and decreasing

Current (I) is generated to create $B_{induced}$ going in (to help the decreasing B)

$\frac{d\Phi_B}{dt}$ (positive or negative B field and negative area vector → positive) so ε (negative)

7. **Induced current in a wire wrapped around iron core**



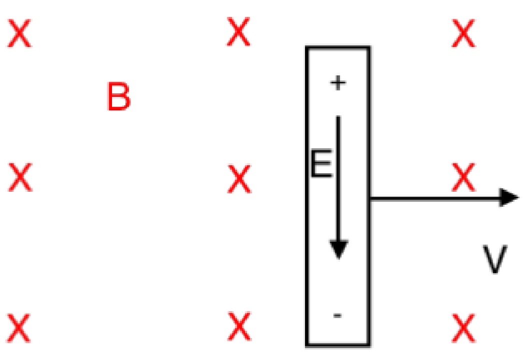
I(t) is increasing so B(t) going left is increasing

Current ($I_{induced}$) is generated to create $B_{induced}$ to oppose B

I(t) is increasing so $\frac{d\Phi_B}{dt}$ (positive) so ε (negative)

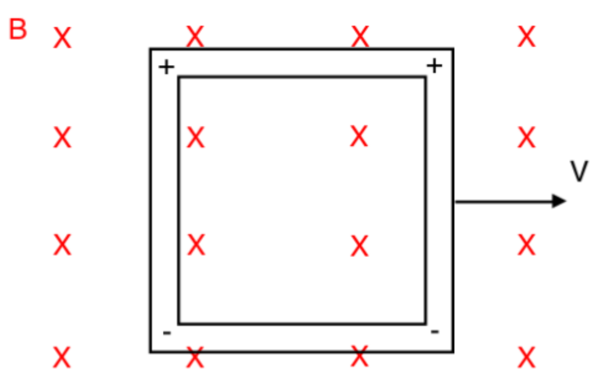
Case Studies for Electromotive Force (EMF due to motion of something)

1. **Metal Bar**



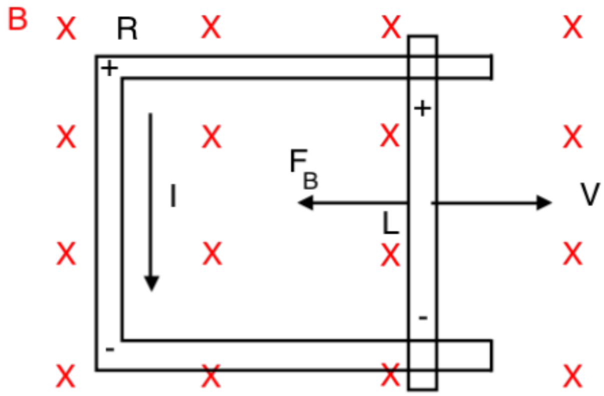
- Metal bar is polarized as charges in the metal bar are moved due to the magnetic field
- There's no continuous flow of charge
- $\mathcal{E} = 0$

2. **Rectangular Loop**



- Metal loop is polarized as charges in the metal loop are moved due to the magnetic field
- $\Phi_B = BL^2$ (constant)
- Since flux is constant, $\mathcal{E} = 0$

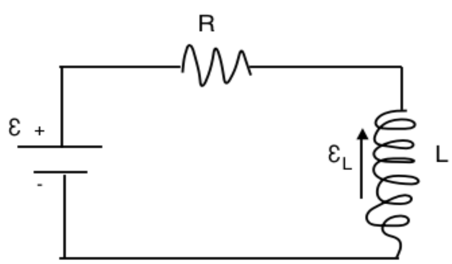
3. **Stationary frame with free moving bar**



- Metal loop is polarized as charges in the metal loop are moved due to the magnetic field
- The moving bar changes flux (because it changes area)
- Φ_B = BLx (x is changing)
- Since flux is changing, $\mathcal{E} \neq 0$
- Current is counter clockwise to create flux coming out to oppose increasing flux going in
- Bar feels a force leftwards, proportional to its speed (bar will eventually stop and no force is felt)

Growth and Decay of RL Circuits (very similar to RC Circuits)

Growth:



Initially (t = 0)**:**
- No current in circuit

Growing (t > 0):
- Current begins to grow
- Current growth creates "back emf" ($\mathcal{E}_L$)
  - $\mathcal{E}_L$ opposes $\mathcal{E}_{battery}$
  - Inductor acts as open switch ($\mathcal{E}_L = \mathcal{E}_{battery}$)
  - All voltage drop at inductor, none at resistor
  - Current growth at fastest rate
- Current begins to level out as it nears the max
  - $\mathcal{E}_L$ decreases as current begins to level out

After Long Time (t → ∞):
- $\mathcal{E}_L \rightarrow 0$
- Inductor acts as highly conductive wire

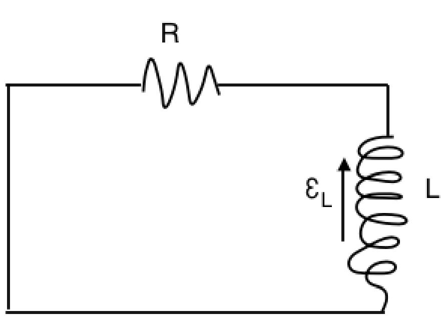
$\tau = \frac{L}{R}$ (time constant)

$I(t) = \frac{\varepsilon}{R}(1 - e^{-\frac{R}{L}t})$ (Current through entire circuit as a function of time)

$\Delta V_R(t) = \varepsilon(1 - e^{-\frac{R}{L}t})$ (Voltage across resistor)

$\varepsilon_L(t) = \varepsilon(e^{-\frac{R}{L}t})$ (Back emf as a function of time)

Decay:



Initially (t = 0 after battery is disconnected after long time)**:**
- Current in circuit is max ($I_o = \frac{\varepsilon}{R}$)

Decaying (t > 0 after battery is disconnected after long time):
- Current begins to decrease

- Voltage across resistor and inductor begins to decrease
- ε_L begins to decrease from ε

$I(t) = \frac{\varepsilon}{R}(e^{-\frac{R}{L}t})$	(Current through entire circuit as a function of time)
$\Delta V_R(t) = \varepsilon(e^{-\frac{R}{L}t})$	(Voltage across resistor)
$\varepsilon_L(t) = -\varepsilon(e^{-\frac{R}{L}t})$	(Back emf as a function of time, negative means its opposing)

Energy in magnetic field in RL circuit:

$u = \frac{B^2}{2\mu_o}$ (energy density, energy stored in magnetic field per volume)

Why Maxwell's Equations?

Background:

How does Maxwell Equations work in empty space? (no charges or current)

Gauss' Law: $\oint E \bullet dA = 0$ ($q_{enc} = 0$)

Gauss' Law for Magnetism: $\oint B \bullet dA = 0$ (No magnetic monopoles)

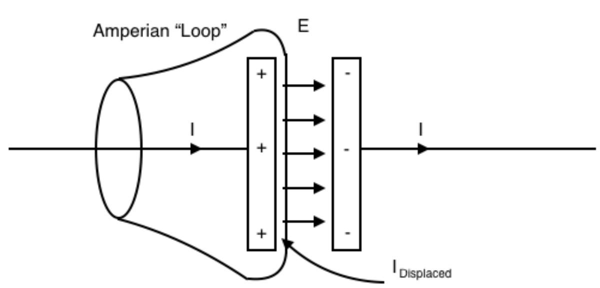
Ampere's Law: $\oint B \bullet ds = 0$ ($I_{total} = 0$)

Faraday's Law: $\oint E \bullet ds = -\frac{d}{dt}\oint B \bullet dA$ (doesn't equal 0)

- Maxwell believed there should be a symmetry between the equations, and therefore didn't think Ampere's Law should equal 0 as well.
- Since Ampere's Law and Faraday's Law had ds in the integral, should they not have similar results?
- Both magnetic fields and E-fields are properties of space so they should have some sort of similarity or relation.

Experiment: Capacitor Paradox

In order to find the true value of Ampere's Law, Maxwell created a capacitor experiment called the Capacitor Paradox.



- Maxwell figured out that the Amperian loop didn't have to be a loop.
- It could be as large or as oddly shaped as it wanted, as long as it was bounded by the area of the loop (essentially keeping that main loop and you can go wherever).
- But what happens when it encapsulates just empty space between the capacitor?

- There's a distribution of charge but no current.
- But we still need to account for those charges between the capacitor plates

Mathematical Proof:

Knowns: $I = \frac{dq}{dt}$ $\Phi_E = \frac{q_{enc}}{\epsilon_o}$ $\oint E \bullet dA = \frac{q_{enc}}{\epsilon_o}$

$\oint E \bullet dA = \frac{q_{enc}}{\epsilon_o}$ [general eq.]

$\frac{d}{dt}\oint E \bullet dA = \frac{dq_{enc}}{dt}\frac{1}{\epsilon_o}$ [take time derivative to get resemblance of current ($\frac{dq_{enc}}{dt}$)]

$\frac{d}{dt}\oint E \bullet dA = I_{displacement}\frac{1}{\epsilon_o}$ [$I_{displacement} = \frac{dq_{enc}}{dt}$]

$I_{displacement} = \epsilon_o \frac{d}{dt}\oint E \bullet dA$

Conclusion:

Ampere-Maxwell's Law: $\oint B \bullet ds = \mu_o I_{tot} + \mu_o \epsilon_o \frac{d}{dt}\oint E \bullet dA$

- This new equation completes the symmetry that Maxwell was looking for.
- It describes how a static B-field creates a changing E-field
- A static E-field creates a changing B-field due to Faraday's Law

After a lot of work, Maxwell eventually proved this with multi-dimensional vector calculus (multivariable), these are the results:

$\frac{\partial^2 E}{\partial x^2} = \mu_o \epsilon_o \frac{\partial^2 E}{\partial t^2}$ $\frac{\partial^2 B}{\partial x^2} = \mu_o \epsilon_o \frac{\partial^2 B}{\partial t^2}$

Eventually, Maxwell obtained the wave formula that describes this self-propagation phenomenon:

u = ??

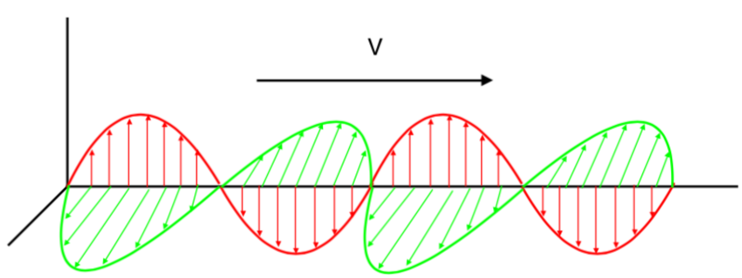
v = speed of wave propagation

$\frac{\partial^2 u}{\partial x^2} - \frac{1}{v^2}\frac{\partial^2 u}{\partial t^2}$

$v = \frac{1}{\sqrt{\mu_o \epsilon_o}} = 2.998 \, x \, 10^8 \, m/s$

REVELATION:

These B-fields and E-fields of empty space are oscillating and self-propagating at the speed of light, proving that light is electromagnetic radiation.



TIPS ON HOW TO SOLVE PROBLEMS:

1. IMPORTANT EQUATIONS/CONSTANTS
   a. $\varepsilon = -\frac{d\Phi_B}{dt} = -\frac{d}{dt}\oint B \bullet dA = \oint E \bullet ds$ (Faraday's Law)
   b. P = $I\Delta v$ (Power Dissipated)
   c. $E = \int Pdt$ (Energy Dissipated)
   d. ε = IR (emf relationship to Current/Resistance)
   e. $\varepsilon_L = -L\frac{dI}{dt}$ (back emf for an inductor)
   f. $L = -\frac{\varepsilon_L}{\frac{dI}{dt}} = N\frac{\Phi_B}{I}$ (inductance, usually given)
   g. $\mu_o = 4\pi \times 10^{-7}$ (Permeability of free space)
   h. $v = \frac{1}{\sqrt{\mu_o \epsilon_o}} = 2.998 \times 10^8\ m/s$ (speed of light/wave propagation)
   i. RL Circuits:
      i. $\tau = \frac{L}{R}$ (time constant)
      ii. $u = \frac{B^2}{2\mu_o}$ (energy density)
      iii. Growth:
         1. $I(t) = \frac{\varepsilon}{R}(1 - e^{-\frac{R}{L}t})$ (Current through entire circuit)
         2. $\Delta V_R(t) = \varepsilon(1 - e^{-\frac{R}{L}t})$ (Voltage across resistor)
         3. $\varepsilon_L(t) = \varepsilon(e^{-\frac{R}{L}t})$ (Back emf as a function of time)
      iv. Decay:
         1. $I(t) = \frac{\varepsilon}{R}(e^{-\frac{R}{L}t})$ (Current through entire circuit)
         2. $\Delta V_R(t) = \varepsilon(e^{-\frac{R}{L}t})$ (Voltage across resistor)
         3. $\varepsilon_L(t) = -\varepsilon(e^{-\frac{R}{L}t})$ (Back emf as a function of time)
   j. $I_{displacement} = \epsilon_o \frac{d}{dt}\oint E \bullet dA$ ("Current" in empty space of capacitor)
   k. $\oint B \bullet ds = \mu_o I_{tot} + \epsilon_o \frac{d}{dt}\oint E \bullet dA$ (Ampere-Maxwell's Law)
2. ε is the same as Δv but ε (emf) is used when dealing with flow in a wire or current while Δv is used for charged particles